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ON THE THEORY OF A FREE AIR JET AND ITS APPLICATION

by

G. N. Abramovich

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
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М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

Page 1.

On the theory of a free air jet and its application.

G. N. Abramovich.

Page 2.

SUMMARY

The present report confined to the problem of a free air jet is divided into three parts.

In the first part the author develops a theory of a free air jet which covers the cases of a plane flow and of an axially symmetrical one (a circular jet). The analysis is based on the Prandtl-Tollmien law of free turbulence. The theory of a free jet allows to determine the form of the flow and the distribution laws for velocity, temperature and those for concentration of gas admixtures both across and along the flow.

In the second part a method of an aerodynamical design of free jets both plane and circular is developed. This method allows to determine the laws of variation of the mean velocity, the discharge and the energy along the free jet.

The third part is confined to the problem of technical applications of the free jet theory; the following problems are considered:

1) The design of an air screen,

2) The resistance of the labyrinth packing of turbo machines (blowers, compressors, pumps etc.),

3) The external resistance of pipe coolers,

4) The deflection of jets, the temperature of which differs from that of the surrounding medium.

The developed theory and the method of calculation applied are in good agreement with the experiment.

The main feature of the present investigation is the use of one experimental constant only namely that of the Prandtl-Tollmien free turbulence theory.

Page 3.

PREFACE.

As the first application/appendix of Prandtl theory of "mixing length" - this basis of the contemporary theories of turbulence, as is known, served free liquid jet in the flooded space. The given mainly by Prandtl himself and the developed further by Tollmien free boundaries theory led to the so/such bright coincidence with the experimental materials, that further development of the idea of "mixing length" with already this fact was provided.

It is necessary to say that in spite of the obvious practical importance of the theory of jet in questions of ventilation and heat engineering and extreme theoretical simplicity of analysis, the engineers did not catch this theory and they continued to remain during the usual hydraulic constructions, supported only by new empirical data. As basic reason this served, apparently certain sketchiness of the theory of the jet of Prandtl-Tollmien, which does not consider the role of the initial diameter of jet so/such important in the short jets.

To G. N. Abramovich belongs the large and serious services of the generalization of the thecry of Prandtl-Tol'min to the case cf the jet of final diameter at the output and the settings and the solutions of the problem abcut the initial section of jet.

After noting the experimental fact of the similarity of the velocity profile in the disturbed part of the initial section to the velocity profile in basic part cf the jet, it extremely in detail and methodically studied the behavicr of jet taking into account to the role of initial section. Using in basic the same mathematical method, as Prandtl-Tollmien, Eng. Abramcvich leads all his linings/calculations to such development, that they make it possible for it to give the complete aercdynamic crew of jet and to stop at the surprisingly interesting and new applications of theory of jet to the series/row of engineering questions.

Are such the given by G. N. Abramovich calculations of resistance of labyrinth seals, bank of tubes and finally the proposed to them theory cf air curtain.

All these questions, until now, underwent no serious investigations. Contemporary ventilation engineer, who carries out questions of "air curtains" greatly much relies on his art and intuition. G. N. Abramovich's wcrk cffers the pcssibility to replace with rational calculaticn the ursteady bases of hydraulic estimations. If the wide circles of our engineers find possible more closely to become acquainted with G. N. Abramovich's works, then they doubtlessly strongly enrich their practical knowledge by the combining theory.

The object/subject of the experiment of G. N. Abramovich is so manifold and complex that it is certainly necessary to even during numerous commercial tests check the correctness of the proposed to them calculations, but already and those checkings which are given by the author for the elongation/extent of his work, they make it necessary to think that author's path is correct.

The theory of air curtain includes a comparatively primitive assumption about the simple superposition of two flows of different structure.

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This, of course, most approximate construction, but is difficult to recommend to the author anything another in this extremely complicated question. If experiment shows sufficient agreement with the calculation, produced on this basis/base, then this approximation/approach to obtain the wide rights of citizenship. In the ideal fluid theory the assumption about the imposition of flows is made constantly.

G. N. Abramovich's work doubtlessly is among distinct works on applied aerodynamics. It it is recessary immediately to publish and to attain wide acceptance in tc the circle of those, who work in the area of ventilation technclcgy. The criticism of these persons will be extremely useful for checking author's basic assumptions, made in his calculations.

Active member of TsAGI, Prof. Dr. Loytsyanskiy

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INTRODUCTION.

It is universally-adcpted engineering aerodynamics to subdivide into twc divisions. The first of them covers the so-called "exterior problem" - the problem of interaction of solid body and air flcw, which flows around about the this bcdy.

The second describes "internal problem" - the problem about air motion in the space, bounded by solid walls (motion in the conduit/manifold).

The division indicated is very convenient and demonstrative, but unfortunately not completely ccmprehensive. The fact is that the contemporary technology, in particular in the latter/last 5-10 years, gave rise to many aerodynamic problems which either partly or whclly cannot be related either to "external" or to the "internal" problems. Are such the questions:

1) the propagation of warm and cold air jets in the production locations;

2) calculation and the design of "air showers" for metallurgical plants;

3) calculation and the construction of the "air curtains", which prevent invasion of cold or contaminated air masses in the industrial buildings;

4) the calculation of open wind-tunnel test section;

5) the calculation of the labyrinth seals of air blowers;

6) resistance of tubular heat exchangers and many others.

The general/common/total feature of the enumerated questions is the fact that they all are directly connected with the problem about the propagation of free jet in the space, filled with the medium of the same physical properties, as the substance of jet. In this case the first four questions it is not completely connected with the problems about relative air motion and solid body. The fifth question can be to the identical degree related both to the "internal problem" and to the "problem about the free jet". The sixth question is connected in the equal measure with all three problems.

The given far incomplete enumeration of the questions of engineering aerodynamics, connected with the problem of free jet, testifies abcut the large urgency of this problem.

It is logical therefore that a whole series of aerodynamicists was occupied by its study. First serious wcrk in this region was published in 1915. Its authcr - Cr.-Engineer T. Truepel - investigated the experimentally velocity fields cf airstream, which escape/ensued from the nczzle irtc large-size airspace[1].

FOOTNOTE [1]. The detailed bibliographical directory of the enumerated works is given at the end of cur article. ENDFOOTNOTE.

The following large experimental investigation of free jet produced in 1921 V. Zimm. The very interesting experimental investigation of air jet is published in 1927 ir the second collector/collection of the Goettingen aerodynamic irstitute.

In 1933, 1934 and 1935 were published, which relate the same question, the works of Ruden, Pcertmann, Couette, Turkus and Predvoditelev.

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In 1936 appeared on the rights/laws of the manuscript the report of

Lyakhovskiy and Syrkin which measured in the jet not only the high-speed/high-velocity, but also the temperature fields.

All these investigations made it possible to come to light/detect/expose experimentally some basic laws governing the course of free jet. However, being purely empirical[1], these works possessed neither completeness nor generality.

FOOTNOTE [1]. Exception/elimination is only Couette's article, in which some laws governing the jet are determined theoretically. ENDFOOTNOTE.

Meanwhile in 1925/26 appeared the remarkable transactions of Prandtl and Tollmien, who developed the theory of free turbulence (Freiturbulenz), which gives the possibility to come to light/detect/expose all basic properties of jet.

Based on it, we developed in 1934 the new method of the aerodynamic design of open wind-tunnel test section[2].

FOOTNOTE [2]. See G. N. Abramovich. Aerodynamics of flow in open wind-tunnel test section. Pt. I and II. Transactions of TsAGI, iss. 223, 236. 1935. ENDFOOTNOTE.

Also based on it is the set-forth below free boundaries theory.

Besides the general theory, this work contains the aerodynamic design of circular and flat/plane free jets, and also the series/row of the engineering applications/appendices of this theory (calculation of trajectory of warm jet, the calculation of air curtain, the calculation of labyrinth seal, the calculation of resistance of heat exchanger).

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Part I.

THEORY OF JET.

§1. Jet structure.

Jet is called free and flooded, if it is unconfined by solid walls and is spread in the space, filled with the medium of the same physical properties, as the substance of jet.

The mechanism of formation/education and propagation of turbulent free jet[1], which escape/ensues from any nozzle into the unlimited flooded space, has the following basic features:

FOOTNOTE [1]. A question about the laminar jet as less urgent/actual, we leave aside. ENDFOOTNOTE.

1. Jet is turbulent; therefore its course is accompanied by mixing current of motion of eddy masses (moles). The latter with their lateral motion fall beyond the limits of jet, they transfer into the contacting with the jet layers of stagnant air their

impulse/momentum/pulse they carry along these layers. In the place of the particles, rejected/thrown out from the jet, into it penetrate the particles of surrounding air which will slow the boundary layers of flow. So is established the exchange of the impulses/momenta/pulses between the jet and the stagnant air, as a result of which the flow mass increases, the width of jet increases, speed at the jet boundaries decreases.

2. Slightly braked particles of active flow together with absorbed particles of surrounding air form turbulent boundary layer of jet whose thickness in direction of course grows/rises. If in the exhaust section of nozzle occurs the even distribution of speeds[2], then in the beginning of jet boundary layer thickness is equal to zero.

FOOTNOTE [2]. That developed/processed in the present work theory of jet is constructed relative to this case. However, will be indicated below the corrections, which should be introduced in the case of nonuniform velocity field in the exhaust section of nozzle. ENDFOOTNOTE.

In this case the boundaries of boundary layer are the divergent surfaces, which intersect at the edge of nozzle (Fig. 1).

From the outer side the boundary layer of jet makes contact with the stagnant air.

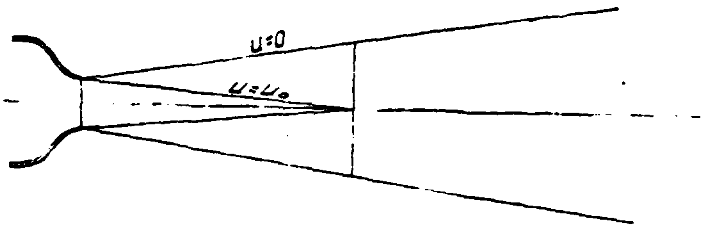

Fig. 1. Propagation of free jet.

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Moreover by outer edge is understood the surface, at all points of which longitudinal (axial) ccmpcnent of the speed is equal to zero[1] (u=0).

FOOTNOTE [1]. Transverse compcnent of speed (V) here cannot be equal to zero, since due to it occurs increase of mass of jet. ENDFOOTNOTE.

From inside the boundary layer converts/transfers into the nucleus of constant velocities. Thus, on the internal boundary of boundary layer speed is equal to the speed cf the undisturbed flow, or, which is the same thing, discharge velccity ($u=u_0$).

In proportion to remcval/distance frcm the nozzle, together with the thickening of the bcundary layer, occurs contraction cf the nucleus of constant velocities. This process leads to the fact that finally the nucleus of the undisturbed flcw disappears entirely.

Subsequently the section cf jet the boundary layer fills already all cross section, stretching up to the axis of flow. Further ercsion of flow is accompanied not cnly by an increase in the width of jet, but also by the incidence/drop in the speed on its axis.

Jet cross-sectional area, in which is completed the liquidation of the nucleus of constant velccities, we will name/call transient. The section, situated between initial and transient jet cross-sectionals area, let us name/call initial. The section, which follows after the transient section, we will name basis. Finally to the point of intersection of external jet boundaries let us appropriate the designation of the pole of jet. The described properties and the configurations of jet clearly demonstrates Fig. 2.

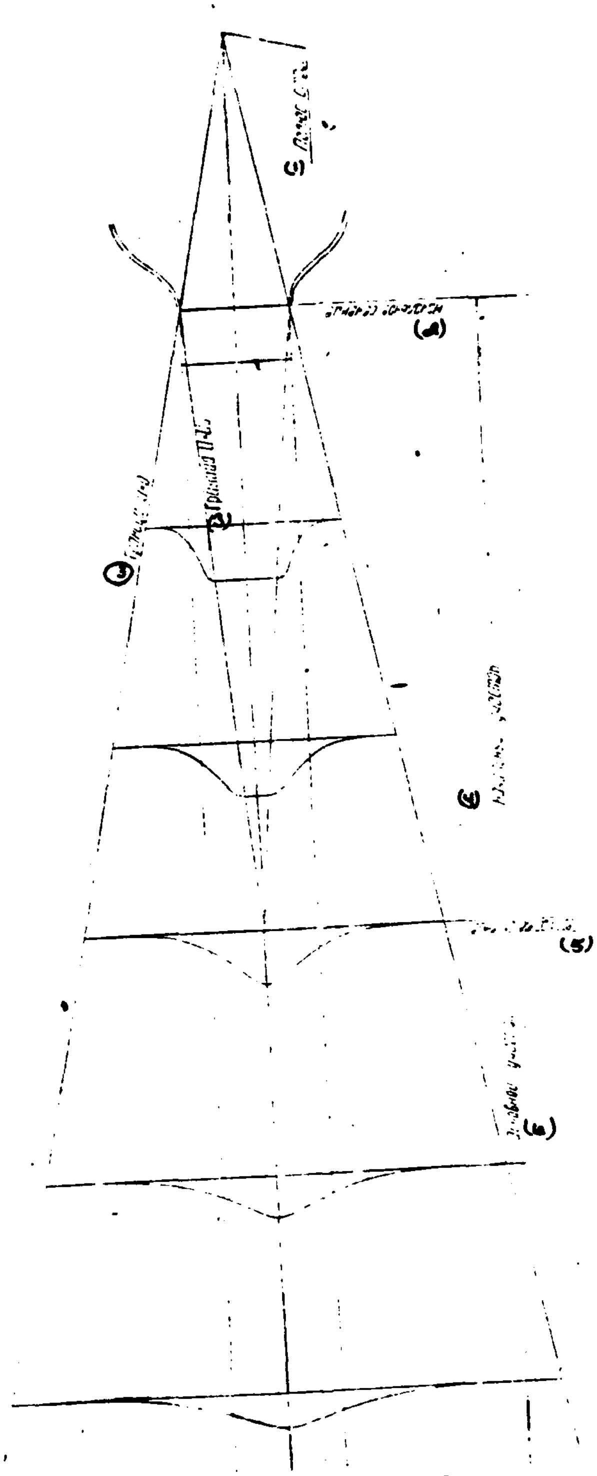

Fig. 2. the diagram of free jet.

Key: (1). Pole of jet. (2). Initial section. (3). Boundary. (4). Initial section. (5). Transient section. (6). Basic section.

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3. Pressure in jet is constant/invariable and it is equal to pressure in surrounding space. Therefore the complete momentum of the mass flow per second of air in all jet cross-sectionals area must remain one and the same.

pressure constancy leads to the fact that in the jet act only the forces of internal friction. The theory of the turbulent flow of Prandtl gives for friction stresses the following dependence:

$$\tau = \rho \cdot l^2 \cdot \left| \frac{\partial u}{\partial y} \right|^2. \qquad (1)$$

Here τ - friction stress;

ρ - air density;

1 - the average value of the mixing length (free path) of the particles of the air in this place of flow;

$\frac{\partial u}{\partial y}$ - the transverse gradient of the longitudinal component of speed.

In application to free jet Prandtl recommends the counting of mixing length by constant for this section of flow and proportional to removal/distance from the beginning of the jet:

$$l = c \cdot x. \quad (2)$$

Here c - constant which proves to be the only experimental constant of the theory of jet.

Comparison of equalities (1) and (2) gives the possibility to compose the formula of friction of the free jet:

$$\tau = \rho \cdot c^2 \cdot x^2 \cdot \left| \frac{\partial u}{\partial y} \right|^2. \quad (3)$$

4. Temperature and air density of jet must be considered equal to temperature and to density of surrounding air. In this case on the jet will not act Archimedes lifting forces and its axis will remain rectilinear[1].

FOOTNOTE [1]. Subsequently will be studied also the case of the jet of variable/alternating density (curvilinear jet). ENDFOOTNOTE.

After becoming acquainted with the basic physical properties of jet, let us pass to the presentation of the theory of jet; in this case the basic and initial sections of jet let us examine separately.

§2. Prerequisites/premises of the study of the basic section of jet.

Fig. 3 gives the distribution curves of the speeds in different sections of the basic section of the circular air jet of Truepel[2].

FOOTNOTE [2]) T. Trüpel. Über die Einwirkung eines Luftstrahles auf die umgebende Luft. Zeitschrift für das gesamte Turbinenwesen. № 5—6. 1915.

ENDFOOTNOTE.

The initial velocity of jet was equal to u_0=87 m/s. A radius of initial jet cross-sectional area was R_0=0.045 m. Velocity fields were removed/taken consecutively/serially to the following distances from the nozzle:

$$S_1 = 0,6\ м;\ S_2 = 0,8\ м;\ S_3 = 1,0\ м;\ S_4 = 1,2\ м;\ S_5 = 1,6\ м.$$

Experiments of Truepel just as the investigation of other authors, testify about the continuous deformation of the high-speed/high-velocity profile/airfoil of jet. The further from the beginning of jet selected the section, that "lower" and "is wider" high-speed/high-velocity profile/airfoil.

To this conclusion/output we come during the construction of velocity fields in the absolute coordinates (u, y). Others and with that much more interesting results are obtained during the deposition of the same fields in the relative coordinates.

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Let us try, for example, tc plot instead of the absolute velocities - u of their relaticn to the speeds cn the axis of jet - $\frac{u}{u_m}$ and instead of the absclute distances from the axis cf jet - y of their relation to the distances from the axis to such points in which the speed is equal to half cf axial - $\frac{y}{y_{\frac{u_m}{2}}}$ (Fig. 4).

The obtained diagram indicates the complete similarity the high-speed/high-velocity cnes of fields in all sections of the basic section of jet.

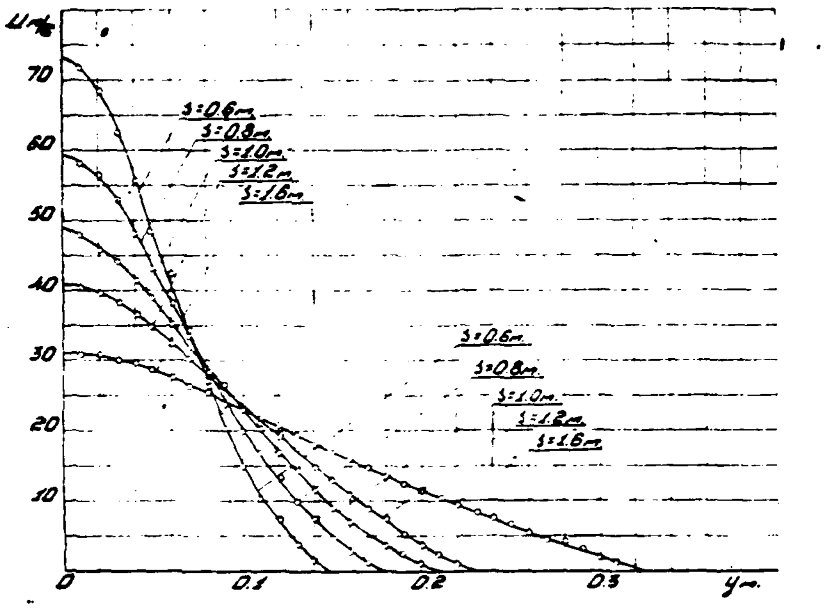

Fig. 3. Velocity fields in different sections of circular jet according to experiments of Truepel.

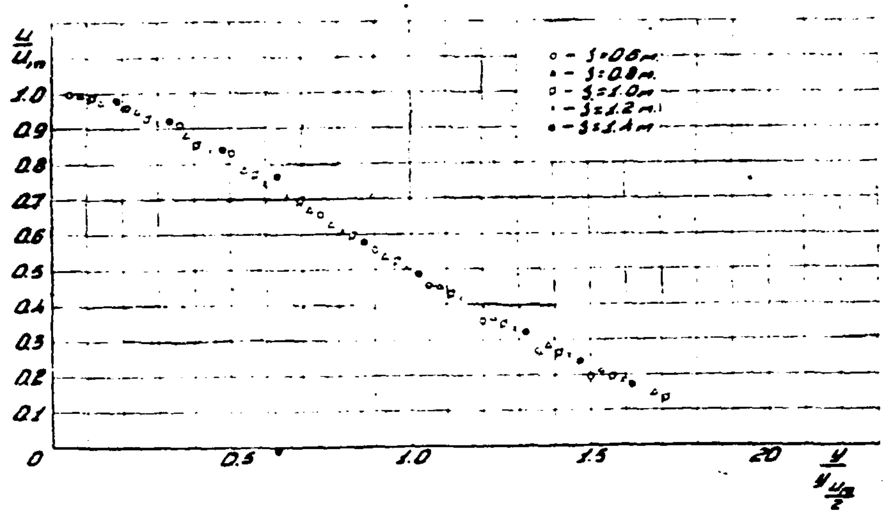

Fig. 4. Relative fields of circular jet.

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This similarity consists in the fact that at congruent points of any two sections of the basic section of jet relative values of speeds and velocity gradients are identical. It is logical that instead of the characteristic length it is possible to take not only $y_{\frac{u_m}{2}}$, but also, for example, the width of jet - b. In this case for congruent points of jet (when $\frac{y_1}{b_1} = \frac{y_2}{b_2}$) we obtain the following characteristic dependences:

$$\left.\begin{aligned} \frac{u_1}{u_{m1}} &= \frac{u_2}{u_{m2}}, \\ \frac{\partial\left(\frac{u}{u_m}\right)_1}{\partial\left(\frac{y}{b}\right)_1} &= \frac{\partial\left(\frac{u}{u_m}\right)_2}{\partial\left(\frac{y}{b}\right)_2}, \end{aligned}\right\} \quad (3a)$$

At congruent points cf kinematically similar turbulent flows must be equal the criteria cf dynamic similarity - Euler's criterion. Therefore

$$\frac{\tau_1}{\rho \cdot u_1^2} = \frac{\tau_2}{\rho \cdot u_2^2}. \quad (4)$$

After replacing τ_1 and τ_2 according to equation (1), we find:

$$\frac{l_1 \cdot \left[\frac{\partial u}{\partial y}\right]_1}{u_1} = \frac{l_2 \cdot \left[\frac{\partial u}{\partial y}\right]_2}{u_2}. \quad (4a)$$

From expressions (3a) we have:

$$\left.\begin{aligned} u_2 &= u_1 \cdot \frac{u_{m2}}{u_{m1}}; \\ \left[\frac{\partial u}{\partial y}\right]_2 &= \left[\frac{\partial u}{\partial y}\right]_1 \cdot \frac{u_{m2}}{u_{m1}} \cdot \frac{b_1}{b_2}. \end{aligned}\right\} \quad (4b)$$

Finally, comparing expressicns (4a) and (4b), we come to the extremely important conclusicn atout equality relative values of mixing length in all sections cf the jet:

$$\frac{l_1}{b_1} = \frac{l_2}{b_2} = \text{idem}. \quad (5)$$

It is interesting to nete that the comparison of equalities (2) and (5) indicates the mutual prcpcrtionality of the width of jet and distance frcm the origin of the coordinates:

$$b = k \cdot x. \quad (6)$$

With x=0 the width of jet is also aqual to zero (b=0). Therefore, placing the origin of coordinates into the pole of jet, we note that the flow in the basic section behaves as if jet is created with the peculiar turbulent source, arranged/located in the beginning of coordinates (in the pole of jet). Using this schematic of course, we will be able to study flow in the entire region of the basic section of jet. However, the initial part of the source which is arranged/located between the pole and the transient section, must be rejected/thrown and the replacement by the initial section of jet.

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The turbulent source indicated, and consequently also the basic section of jet, possesses one very intrinsic property which consists in the fact that along any rectilinear ray/beam, directed from the pole and lying within the limits of jet, the relative speed of flow retains the constant value:

$$\frac{u}{u_m} = f\left(\frac{y}{x}\right). \qquad (7)$$

In other words, congruent points of the basic section of jet lie/rest on one ray/beam (Fig. 5).

This property not difficult to reveal/detect, if to compare the geometric determination of congruent points (y/b=const) with equation 6 (b/x=const).

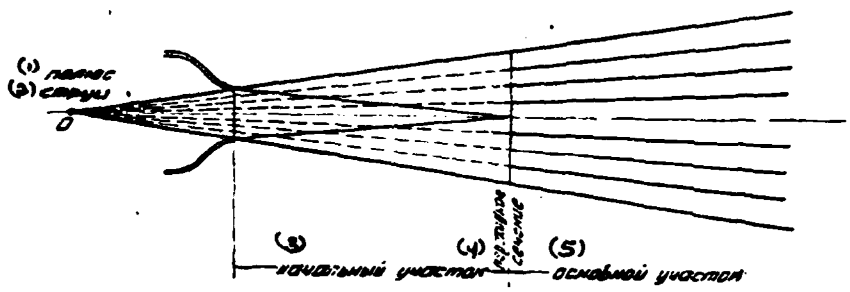

Fig. 5. Rays/beams of the ccnstant velocities of the basic secticn of free jet.

Key: (1). pole. (2). jet. (3). Initial section. (4). Transient section. (5). Basic section.

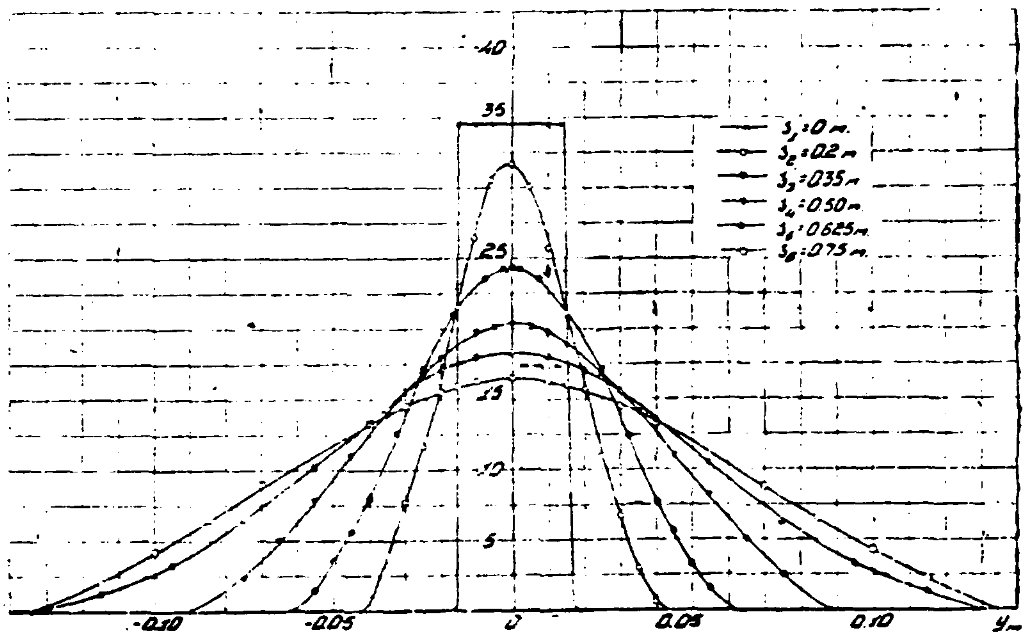

Fig. 6. Velocity fields in different sections of plane of jet according to experiments of Foertmann.

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Being based on it and taking into account that the speed on the axis can depend only on abscissa $[u_m = \Theta(x)]$, we obtain in general form the law of the speeds of the basic section of the jet:

$$u = \Theta(x) \cdot f\left(\frac{y}{x}\right).$$

The obtained conclusion/output, which rest like the velocity fields in all sections, basic section, are valid not only for the circular jet. In the equal measure they relate also to the slot jet.

In order to be convinced cf this, it suffices to examine the results of experiments cf Foertmann[1].

FOOTNOTE [1]. E. Fortmann. „Über turbulente Strahlausbreitung". Ingenieur-Archiv Bd. V. H. [illegible] 1934 г.

ENDFOOTNOTE.

Last was studied the velocity fields of the slot jet, which escape/ensued from the opening/aperture in narrow fitting with a width of 0.03 m and by length 0.65 m. This jet velocity was equal to 35 m/s. In Fig. 6 are plotted the velocity fields, taken/removed in jet cross-sectionals area which are distant from the nozzle up tc the distances:

$$S_1 = 0\ м;\ S_2 = 0.20\ м;\ S_3 = 0,35\ м;\ S_4 = 0,50\ м;\ S_5 = 0,625\ м;\ S = 0,7?\ м.$$

Being they are transferred into the relative coordinates (the same as for experiments of Trüpel) these fields, just as in the case of slot jet, they prove tc be similar (Fig. 7).

Thus, in the main section cf slot jet (as in the circular jet) flow also as is created with the turbulent source, placed in the pole of jet. The corresponding turbulent sources (circular source - point and flat/plane source - line) are studied by W. Tollmien in his remarkable research on free turbulence[2].

FOOTNOTE [2]. W. Tollmien. Berechnung turbulenter Ausbreitungsvorgänge. Z. A. M. M. 1926; Bd. 6 Heft 6.

ENDFOOTNOTE.

To the selection/analysis of the theory of Tollmien on basis of which is constructed aerodynamics of the basic section cf jet, we will pass subsequently the paragraph cf cur wcrk.

In order to make theory cf Tollmien as widely available as possible, we will set fcrth it ir scmewhat more detail than this made author himself.

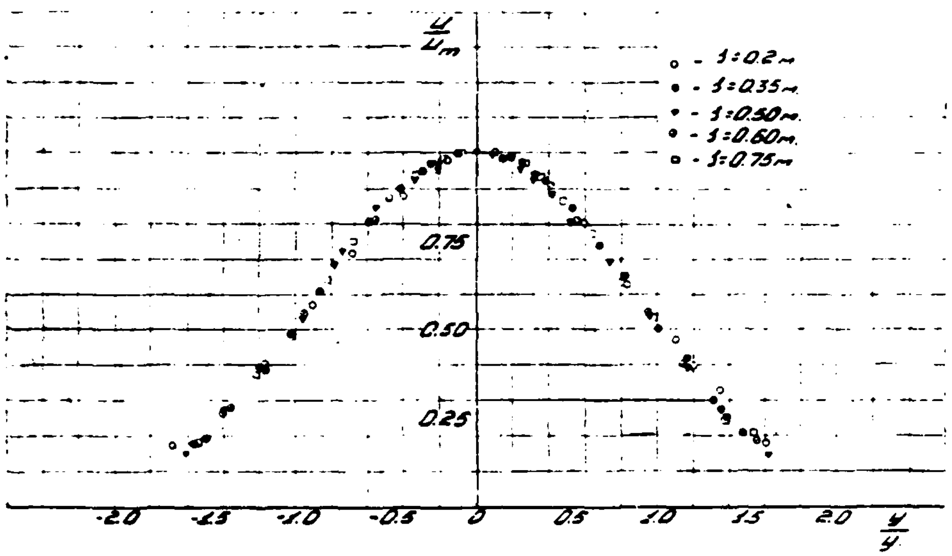

Fig. 7. The velocity fields of slot jet in the relative coordinates.

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§3. Turbulent source of slot jet.

Let us examine the free flow which is formed during the discharge of the air through the narrow rectilinear slot in the wall (Fig. 8). Axis x consistent with the axis of jet, y axis it is directed perpendicularly to slot, the origin of coordinates let us place directly into the slot.

Pressure in the jet is constant/invariable; therefore the complete momentum of mass flow per second must remain the constant:

$$\int_{-\infty}^{+\infty} u^2 \cdot dy = \text{const.}$$

For studying the jet we will use coordinates - x; $\eta = \frac{y}{x}$.

The longitudinal velocity in this coordinate system is equal [see equality (8)]:

$$u = \Theta(x) \cdot f(\eta), \tag{9}$$

whence

$$\Theta^2(x) \cdot x \cdot \int_{-\infty}^{+\infty} f^2(\eta) \cdot d\eta = \text{const}.$$

In turn:

$$\int_{-\infty}^{+\infty} f^2(\eta) \cdot d\eta = \text{const}.$$

Therefore speed on the axis of jet (u_m) is inversely proporticnal to square root of the abscissa:

$$u_m = \Theta(x) = \frac{\text{const}}{\sqrt{x}} = \frac{n}{\sqrt{x}}. \tag{10}$$

However, the longitudinal speed at the arbitrary point of jet comprises:

$$u = \frac{n}{\sqrt{x}} \cdot f(\eta). \tag{11}$$

Let us introduce the function of current ψ.

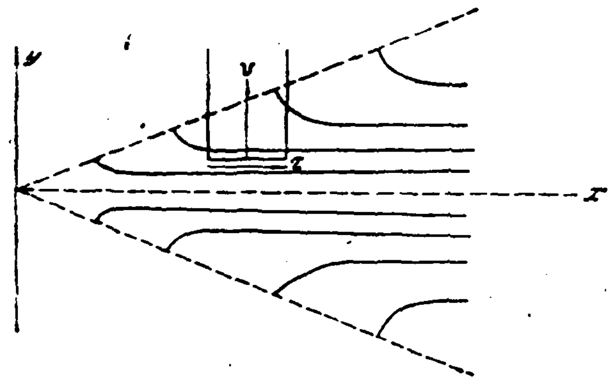

Fig. 8. Turbulent source of slot jet.

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As is known, its partial derivatives are equal to the longitudinal and transverse components of the speed:

$$u=\frac{\partial\psi}{\partial y},$$
$$v=-\frac{\partial\psi}{\partial x},$$

The function of current will be determined from expression (11):

$$\psi=\int u\cdot dy=n\cdot\sqrt{\bar{x}}\cdot\int f(\eta)\cdot d\eta.$$

After accepting the following designation:

$$\int f(\eta)\cdot d\eta=F(\eta).$$

we will obtain new expressions for functioning the current and component of speed.

The function of current is equal to:

$$\psi = n \cdot \sqrt{x} \cdot F(\eta). \qquad (12)$$

Longitudinal component of the speed:

$$u = \frac{n}{\sqrt{x}} \cdot F'(\eta). \qquad (13)$$

Transverse component of the speed:

$$v = -\frac{\partial \psi}{\partial x} = \frac{n}{\sqrt{x}} \cdot \left[\eta \cdot F'(\eta) - \frac{1}{2} \cdot F(\eta)\right]. \qquad (14)$$

The basic problem of the investigation of flat/plane turbulent source must consist of finding of function $F(\eta)$. The solution of this problem proves to be possible with the aid of the theorem of momentum. Let us isolate for this purpose in Fig. 8 control surface and let us examine momentum balance within it. Through the lower part of this surface is transferred second-by-second certain momentum:

$$\rho \cdot u \cdot v \cdot dx$$ [1].

FOOTNOTE [1]. The length of surface along the perpendicular to the plane of drawing we consider equal to unity. ENDFOOTNOTE.

Along other side of control surface is observed a change in the momentum:

$$-d \int_y^\infty \rho \cdot u^2 \cdot dy.$$

As force serves shearing turbulent stress [see equation (3)

$$\tau = \rho \cdot c^2 \cdot x^2 \cdot \left[\frac{\partial u}{\partial y}\right]^2.$$

After some elementary conversions we obtain momentum equation in the following form:

$$u \cdot v + \frac{\partial}{\partial x} \int_x^{\infty} u^2 \cdot dy + c^2 \cdot x^2 \cdot \left[\frac{\partial u}{\partial y} \right]^2 = 0. \qquad (15)$$

We utilize now dependences (13) and (14) for the modification of equation (15), attempting to turn it into the differential equation of function $F(\eta)$.

We will obtain:

$$u \cdot v = \frac{n^2}{x} \left\{ [F'(\eta)]^2 \cdot \eta - \frac{1}{2} \cdot F'(\eta) \cdot F(\eta) \right\};$$

$$\frac{\partial}{\partial x} \int u^2 \cdot dy = -\frac{n^2}{x} \cdot [F'(\eta)]^2 \cdot \eta;$$

$$c^2 \cdot x^2 \cdot \left[\frac{du}{dy} \right]^2 = \frac{n^2}{x} \cdot c^2 \cdot [F''(\eta)]^2.$$

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After the substitution of the obtained equalities in equation (15), we come to the differential the second order equation:

$$2 \cdot c^2 \cdot [F''(\eta)]^2 = F(\eta) \cdot F'(\eta). \qquad (16)$$

Into equation (16) enters the experimental constant c. This bears out the fact that in the selected system of coordinates $(x; \eta)$ the unknown functions depend on jet structure. It is not difficult to show that by the simple modification of ordinate it is possible to eliminate the effect of jet structure on function $F(\eta)$. For this instead of η let us introduce the new relative ordinate:

$$\varphi = \frac{\eta}{a}, \qquad (17)$$

where

$$a = \sqrt{2 \cdot c^2}$$

In the new system of coordinates $(x; \varphi)$ the components of the speed will accept the following form:

$$u = \frac{n}{\sqrt{a \cdot x}} \cdot F'(\varphi) = u_m \cdot F'(\varphi); \qquad (18)$$

$$v = -\frac{a \cdot n}{\sqrt{a \cdot x}} \cdot \left[\varphi \cdot F'(\varphi) - \frac{1}{2} \cdot F(\varphi)\right], \qquad (19)$$

because of which the fundamental differential equation will be written as follows:

$$[F''(\varphi)]^2 = F(\varphi) \cdot F'(\varphi). \qquad (20)$$

Before beginning the solution of equation (20), let us lower its order. Let us introduce for this dependant variable

to what it corresponds

$$z = \ln F(\varphi),$$
$$F(\varphi) = e^z.$$

The substitution of this variable/alternating in the initial equation gives the following result:

$$[z'' + (z')^2]^2 = z'.$$

After one additional replacement $-z' = Z$, we obtain differential first-order equation:

$$Z' = -Z^2 - \sqrt{Z}. \qquad (21)$$

Before value $\sqrt{Z}$ we supplied minus sign, since from the physical considerations it follows that sum $Z' + Z^2 \leqslant 0$. Actually:

$$Z = z';$$
$$Z' = z'';$$
$$z = \ln |F(\varphi)|;$$
$$z' = \frac{F'(\varphi)}{F(\varphi)};$$
$$z'' = \frac{F''(\varphi)\cdot F(\varphi) - [F'(\varphi)]^2}{[F(\varphi)]^2}.$$

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Hence

$$Z' + Z^2 = z'' + (z')^2 = \frac{F''(\varphi)}{F(\varphi)};$$

but

$$F''(\varphi) \sim \frac{\partial u}{\partial y} \text{ and } F(\varphi) \sim \int u \cdot dy.$$

In other words, F" (φ) corresponds to the transverse gradient of speed, whereas F(φ) corresponds to expenditure/consumption. Expenditure/consumption is positive value. The at the same time transverse gradient of the speed (Fig. 3 and 6) at all points of jet cross-sectional area is lower than zero and only on the axis and at the outer edge it is equal to zero.

Thus:

$$\frac{F''(\varphi)}{F(\varphi)} = Z' + Z^2 \leqslant 0.$$

The solution of equation (21) does not represent special labor/work, since variable/alternating in it are divided:

$$\varphi = c - \int \frac{dZ}{Z^2 + 1\ Z}. \qquad (21a)$$

Using the general method of integrating the rational integral functions, we find this integral:

$$\varphi = c - \frac{2}{3}\cdot\left\{\ln(\sqrt{Z}+1) - \ln\left[\sqrt{(Z-\sqrt{Z}+1)}\right] + \sqrt{3}\cdot\operatorname{arctg}\frac{2\sqrt{Z}-1}{\sqrt{3}}\right\}. \qquad (22)$$

Let us examine the new boundary conditions by which it must satisfy obtained integral (22). The first boundary condition lies in the fact that on the axis of jet - with $\phi=0$ - the transverse component of speed (v) is equal to zero, and relative value of longitudinal velocity $\left(\frac{u}{u_m}\right)$ is equal to one.

Because of this with $\phi=0$:

$$F(\varphi) = e^z = 0,$$

$$F'(\varphi) = \frac{u}{u_m} = z'\cdot e^z = 1.$$

Whence

$$Z = z' = \;. \qquad (22a)$$

The second boundary condition consists in the fact that on the external jet boundary - when $\varphi = \varphi_{\text{гр}}$ - the transversing speed does not disappear, while the longitudinal velocity is equal to zero. Therefore when $\varphi = \varphi_{\text{гр}}$:

$$F(\varphi) = e^z = 0,$$

$$F'(\varphi) = z'\cdot e^z = 0.$$

Hence

$$Z = z' = 0. \qquad (22b)$$

From condition (22a) we compute the constant of the integration:

$$c=0+\frac{2\cdot\sqrt{3}}{3}\cdot\frac{\pi}{2}=\frac{\pi}{\sqrt{3}}=1{,}81.$$

Condition (22b) gives the possibility to determine the relative ordinate of jet boundary:

$$\varphi_{\text{гр}}=\frac{\pi}{\sqrt{3}}-\frac{2}{3}\cdot\sqrt{3}\cdot\operatorname{arctg}\left(-\frac{1}{\sqrt{3}}\right)=\frac{4\cdot\pi}{3\cdot\sqrt{3}}=2{,}412.$$

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Equation (22) is utilized for finding the values of the second auxiliary function - $Z=z'$. The values of the first auxiliary and basic functions (z and $F(\varphi)=e^{z}$) are determined with the aid of the quadratures and taking the logarithm from the following two equalities:

$$\left.\begin{aligned} Z&=z_0+\int_{\varphi_0}^{\varphi} Z\cdot d\varphi;\\ z&=\ln[F(\varphi)].\end{aligned}\right\} \qquad (23)$$

Here z - unknown value of function.

φ - corresponding value of argument (relative ordinate).

z_0 - known value of function at any point with the ordinate φ_0.

Above it was shown that near the axis of jet Z approaches positive infinity, and z - to negative infinity (when $\varphi=0; e^{z}=0$):

$$Z = z' \to \infty,$$
$$z \to -\infty.$$

This fact indicates the inadequacy of equation (22) and equalities (23) near the axis of jet. Let us try therefore to find new solution, suitable in stand regions. Proceed we will be as before from differential equation (21):

$$\frac{dZ}{d\varphi} = -Z^{2} - 1\,Z.$$

With $\varphi \to 0$, when $Z \to \infty$, equation (21) can be strongly simplified. For this should be disregarded/neglected low value $1\,Z$.

Thus, we obtain:

$$\frac{dZ}{d\varphi} = -Z^{2}. \qquad (24)$$

Whence

$$Z = z' = \frac{1}{\varphi} \qquad (25)$$

and

$$z = -\ln\varphi + c_{1}.$$

Integration constant can be determined from those considerations, that on the axis of jet the relative speed is equal to unity:

$$\frac{u}{u_m} = F'(0) = z' \cdot e^{z} = 1,$$

otherwise

$$\frac{1}{\varphi} \cdot e^{\ln\varphi - c_1} = \frac{e^{\ln\varphi}}{\varphi} \cdot e^{c_1} = 1.$$

But

$$e^{\ln\varphi} = \varphi^{1}).$$

thanks to which

$$c_1 = 0.$$

FOOTNOTE [1]. Of this it is not difficult to be convinced by means of the logarithmic operation. ENDFCCTNOTE.

Thus, the first auxiliary functicn, which we introduced for a decrease in the order cf differential equation (20), approaches lnφ:

$$z \to \ln \varphi.$$

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Generally z it is possible tc present in the form of the series/row, first member of which is lnφ:

$$z = \ln \varphi + \ldots, \tag{26}$$

hence

$$z' = \frac{1}{\varphi} + \ldots \tag{27}$$

The most immediate task of the research is finding subsequent members of series/row (27), and then series/row (26). Is solved this problem by the method of successive approximations.

We assume/set in the first approximation:

$$Z = z' = \frac{1}{\varphi} + A\varphi^b$$

and we produce the substitution of this expression in differential equation (21):

$$-\frac{1}{\varphi^2}+A_1\cdot t_1\cdot\varphi^{t_1-1}=-\frac{1}{\varphi^2}-2\cdot A_1\cdot\varphi^{t_1-1}-A_1^2\cdot\varphi^{2t_1}-\sqrt{\frac{1}{\varphi}+A_1\cdot\varphi^{t_1}}\cdot$$

Disregarding components/terms/addends with highest degrees ($A_1^2\cdot\varphi^{2t_1}$ and $A_1\cdot\varphi^{t_1}$), we obtain:

$$A_1(t_1+2)\cdot\varphi^{t_1-1}=-\varphi^{-\frac{1}{2}}.$$

Equalizing between themselves first exponents, and then coefficients in two parts of this equality, we find the second term of series/row (27):

$$A_1\cdot\varphi^{t_1}=-0{,}4\cdot\sqrt{\varphi}.$$

For finding the third term of series/row, we resort to the substitution into equation (21) of function Z, undertaken in the second approximation/approach:

$$Z=z'=\frac{1}{\varphi}-0{,}4\cdot\sqrt{\varphi}-A_2\cdot\varphi^{t_2}.$$

From this in a described above manner we obtain:

$$A_2\cdot\varphi^{t_2}=0{,}01\cdot\varphi^2.$$

Continuing calculations in the same order, it is not difficult to find any quantity of terms of series/row. For our calculations it is possible to be restricted by three members.

Thus, we have:

$$z'=\frac{1}{\varphi}-0{,}4\cdot\sqrt{\varphi}-0{,}01\cdot\varphi^2-\dots \qquad (27a)$$

Whence

$$z=\ln\varphi-\frac{0{,}8}{3}\cdot\varphi\sqrt{\varphi}-\frac{0{,}01}{3}\cdot\varphi^3+\dots \qquad (28a)$$

Asymptotic approximations/approaches (27a) and (28a) give the possibility to calculate the values of the functions:

$$z'(\varphi);\ z(\varphi);\ F(\varphi)=e^{z}\ \overset{(1)}{\text{И}}\ F'(\varphi)=z'e^{z}$$

Key: (1). and.

at points with the low values of argument φ.

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At other points of the cross section of slot jet should be utilized equation (22) and equalities (23). After producing the calculations indicated and utilizing formulas (18) and (19):

$$\frac{u}{u_m} = F'(\varphi); \tag{29}$$

$$\frac{1}{a} \cdot \frac{v}{u_m} = \varphi \cdot F'(\varphi) - \frac{1}{2} \cdot F(\varphi), \tag{30}$$

we compose Tables 1 and Fig. cf 9-10 longitudinal and transversing speeds in the cross section of the basic section of slot jet.

Table 1 and Fig. 9 and 10 speeds u and v depict in the portions of the velocity on the axis of jet (u_m). In turn, the axial velocity of slot jet is the function of polar distance of the jet [see equality (18)]:

$$u_m = \frac{n}{\sqrt{ax}}. \tag{31}$$

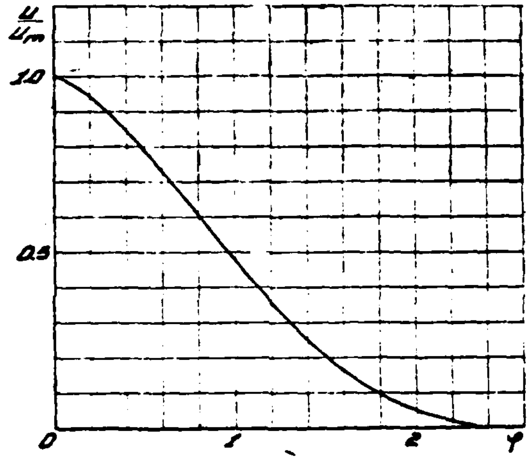

Fig. 9.

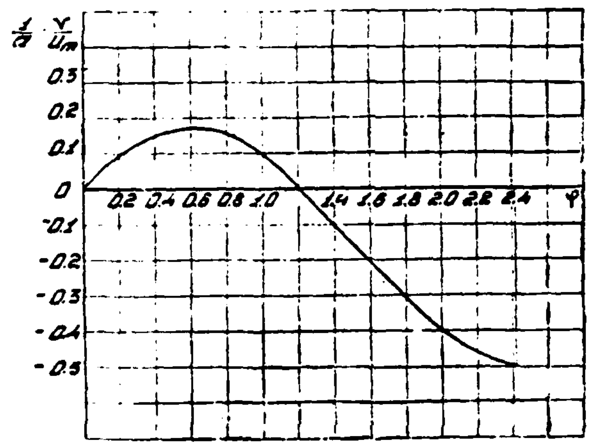

Fig. 10.

Fig. 9. Distribution of longitudinal velocities in section of slot jet.

Fig. 10. Distribution of transversing speeds in section of slot jet.

Table 1.

φ	$\frac{u}{u_m} = F(\varphi)$	$\frac{1}{a} \cdot \frac{V}{u_m}$
0	1,000	0
0,1	0,979	0.049
0,2	0,940	0,091
0,3	0,897	0,120
0,4	0,842	0,151
0,5	0,782	0.166
0,6	0,721	0,168
0,7	0,660	0,166
0,8	0,604	0,151
0,9	0,538	0,120
1,0	0,474	0,091
1,1	0,411	0,049
1,2	0,357	0
1,3	0.300	— 0,056
1,4	0,249	— 0,099
1,5	0,200	— 0,160
1,6	0,165	— 0.212
1,7	0,125	— 0,260
1,8	0,095	— 0,318
1,9	0,067	— 0,356
2,0	0,046	— 0,402
2,1	0,030	— 0,440
2,2	0,020	— 0.469
2,3	0,009	— 0,490
2,4	0	— 0,498

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Let us give to expression (31) somewhat different form. For this again we will use the constancy of momentum in the free slot jet:

$$\rho \int_0^{y_{rp}} u^2 \cdot dy = \rho \cdot u_0^2 \cdot b_0 = \text{const.}$$

Here y_{rp} - ordinate of jet boundary; ρ - air density; b_0 - half-width of initial jet cross-sectional area; u_0 - discharge velocity.

After the replacement of the variable/alternating (instead of y we introduce ϕ=y/ax) we obtain:

$$\frac{a\cdot x}{b_0}\cdot\frac{u_m^2}{u_0^2}\cdot\int_0^{\varphi_{rp}=2,4}\frac{u^2}{u_m^2}\cdot d\varphi=1$$

or otherwise

$$\frac{u_m}{u_0}=\frac{1}{\sqrt{\frac{a\cdot x}{b_0}}\cdot\sqrt{\int_0^{2,4}[F'(\varphi)]^2\cdot d\varphi}}.$$

Using Tables 1, we compute using the method of trapezoids the integral:

$$\int_0^{2,4}[F'(\varphi)]^2 d\varphi=0,685.$$

Replacing integral by its numerical value, we reduce to final form the formula of the axial velocity[1] of the slot jet:

$$\frac{u_m}{u_0}=\frac{1,2}{\sqrt{\frac{a\cdot x}{b_0}}}. \qquad (32)$$

FOOTNOTE [1]. The axial velocity is measured here in the portions of the discharge velocity. ENDFOOTNOTE.

In the beginning of the basic section of slot jet - in the transient section - the axial velocity is equal to discharge velocity:

$$u_m = u_0.$$

Hence we find the relative abscissa (relative polar distance) of the transient section of the slot jet:

$$\frac{a \cdot L_0}{b_0} = 1{,}44. \qquad (33)$$

All obtained functions of the turbulent source of slot jet are suitable only in the region of the basic section of the jet when

$$x \geqslant L_0. \qquad (34$$

Participating in all dependences coefficient a is determined from experiments. To a question about the selection of its values in different cases will be dedicated the special paragraph of this work.

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§4. Turbulent source of circular jet.

In the present paragraph we will become acquainted with the theory of the formation/education of free jet during the discharge of air from the circular opening/aperture of very small sizes/dimensions (Fig. 11).

Section jet-circle; therefore the law of conservation of momentum will be written as follows:

$$2 \cdot \pi \cdot \int_{-x}^{+x} u^2 \cdot y \cdot dy = \text{const}.$$

We select for the investigation the coordinate system with the axes:

$$x;\ \eta=\frac{y}{x}.$$

In this coordinate system the condition of the constancy of momentum assumes somewhat different form:

$$2\pi\cdot u_m^2\cdot x^2\cdot\int_{-x}^{+x}\frac{u^2}{u_m^2}\cdot\eta\cdot d\eta=$$
$$=\text{const},$$

whence

$$u_m^2\cdot x^2\cdot\int_{-x}^{+x}f^2(\eta)\cdot\eta\cdot d\eta=\text{const}.$$

However, $\int f^2(\eta)\cdot\eta\cdot d\eta$ is a constant value, due to what the law of a change in the axial velocity acquires the hyperbolic form:

$$u_m=\frac{m}{x}. \qquad (35)$$

Dependence (35) gives the possibility to obtain the formula of the longitudinal component of the speed:

$$u=u_m\cdot f(\eta)=\frac{m}{x}\cdot f(\eta). \qquad (36)$$

The longitudinal and transverse components of the speed in the flow, symmetrical relative to rotational axis (x axis), can be as follows expressed by means of the function of the current:

$$u=\frac{1}{y}\cdot\frac{\partial\psi}{\partial y}$$
$$v=-\frac{1}{y}\cdot\frac{\partial\psi}{\partial x}.$$

Hence

$$\psi=\int u\cdot y\cdot dy=m\cdot x\cdot\int f(\eta)\cdot\eta\cdot d\eta.$$

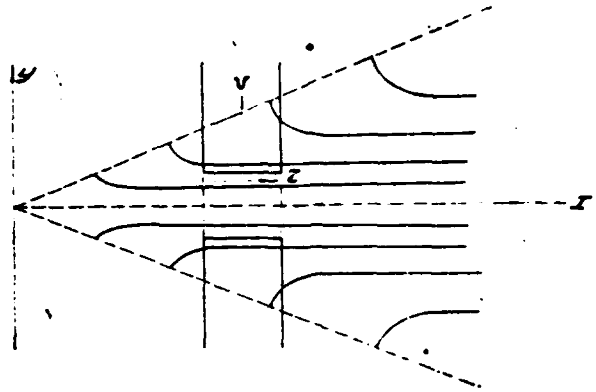

Fig. 11. Turbulent source of circular jet.

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Let us introduce the designation:

$$\int f(\eta)\cdot\eta\cdot d\eta = F(\eta)$$

and let us recompose the formulas of the function of current, longitudinal velocity and transversing speed:

$$\psi = m\cdot x\cdot F(\eta); \quad (37)$$

$$u = \frac{m}{x}\cdot\frac{F'(\eta)}{\eta}; \quad (38)$$

$$v = -\frac{1}{y}\cdot\frac{\partial\psi}{\partial x} = \frac{m}{x}\cdot\left[F'(\eta) - \frac{F(\eta)}{\eta}\right]. \quad (39)$$

The basic problem of the investigation of the turbulent source of circular jet consists of finding of function F(η) and its derivative F'(η).

The solution of this problem can be obtained with the aid of the theorem of momentum[1].

FOOTNOTE [1]. Just as in the case of slot jet. ENDFOOTNOTE.

Let us isolate for this purpose in Fig. 11 symmetrical relative to x axis control surface and let us compose for it momentum balance. Through the lower part of control surface, which has area - $2\cdot\pi\cdot y\cdot dx$ is transferred second-by-second the momentum:

$$2\cdot\pi\cdot\rho\cdot u\cdot v\cdot y\cdot dx.$$

Within the control surface occurs a change in the momentum per second:

$$-2\pi\rho\cdot d\int_{y}^{\infty} u^2\cdot y\cdot dy.$$

As a result of changing the momentum appears the tangential force:

$$\tau\cdot 2\cdot\pi\cdot y\cdot dx=\rho\cdot c^2\cdot x^2\cdot\left[\frac{\partial u}{\partial y}\right]^2\cdot 2\cdot\pi\cdot y\cdot dx.$$

Utilizing the enumerated three factors, we compile an equation momenta:

$$u\cdot v+\frac{1}{y}\cdot\frac{\partial}{\partial x}\int_{y}^{\infty} u^2\cdot y\cdot dy+c^2\cdot x^2\cdot\left[\frac{\partial u}{\partial y}\right]^2=0. \qquad (40)$$

Equalities (38) and (39) give the possibility to convert equation (40) into the differential equation of function $F(\varphi)$. Actually:

$$u\cdot v = \frac{m^2}{x^2}\cdot\left[\frac{[F'(\eta)]^2}{\eta} - \frac{F(\eta)\cdot F'(\eta)}{\eta^2}\right];$$

$$\frac{1}{y}\cdot\frac{\partial}{\partial x}\int u^2\cdot y\cdot dy = -\frac{m^2}{x^2}\cdot\frac{[F'(\eta)]^2}{\eta};$$

$$c^2\cdot x^2\cdot\left[\frac{\partial u}{\partial y}\right]^2 = \frac{m^2\cdot c^2}{x^2}\cdot\left[F''(\eta) - \frac{F'(\eta)}{\eta}\right]^2\cdot\frac{1}{\eta^2}$$

which after substitution in equation (40) gives:

$$c^2\cdot\left[F''(\eta) - \frac{F'(\eta)}{\eta}\right]' = F(\eta)\cdot F'(\eta). \qquad (41)$$

From the obtained differential equation it is possible to exclude the experimental constant c^2.

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For this it suffices to change the coordinate system, after switching over from the coordinates

$$x;\ \eta = \frac{y}{x}$$

to the coordinates

$$x;\ \varphi = \frac{y}{a\cdot x},$$

where

$$a = \sqrt[3]{2c^2}.$$

In the new coordinate system the components of the speed will accept the following form:

$$u = u_m\cdot\frac{F'(\varphi)}{\varphi} = \frac{m}{a\cdot x}\cdot\frac{F'(\varphi)}{\varphi} \qquad (42)$$

$$v = \frac{m}{x}\cdot\left[F'(\varphi) - \frac{F(\varphi)}{\varphi}\right]. \qquad (43)$$

Utilizing new formulas of speeds, we have the capability to give to fundamental differential equation the new form:

$$F''(\varphi) - \frac{F'(\varphi)^2}{\varphi} = F'(\varphi) \cdot F(\varphi). \qquad (44)$$

Attempting to lower the order of differential equation (44), let us introduce new dependant variable z=ln [F(φ)], to what corresponds F(φ)=e^z. After the substitution of new variable/alternating we obtain:

$$z'' + (z')^2 - \frac{z'^2}{\varphi} = z'.$$

If we assume z'=Z, then is formed differential first-order equation:

$$Z' = \frac{Z}{\varphi} - Z^2 - \sqrt{Z}. \qquad (45)$$

With component/term/addend $\sqrt{Z}$ just as in the case of slot jet[1], is accepted negative sign, which corresponds to the sign of the transverse gradient of speed $\left(\frac{\partial u}{\partial y} < 0\right)$.

FOOTNOTE [1]. See equation (21). ENDFOOTNOTE.

Let us examine the necessary for the integration boundary conditions.

1. On axis of jet where $\phi=0$, must disappear transverse component of speed:

$$v=0.$$

Equality (43) it leads in this case to the conditions:

$$\left.\begin{array}{l} F(0)=e^{z}=0, \\ z=\ln[F(\varphi)]=-\infty. \end{array}\right\} \qquad (45a)$$

Furthermore, on the axis of jet relative value of longitudinal velocity is equal to one:

$$\frac{u}{u_m}=1.$$

Whence (for $\phi=0$)

$$\frac{F'(\varphi)}{\varphi}=\frac{z'\cdot e^{z}}{\varphi}=1. \qquad (45b)$$

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2. On jet boundary, when $\varphi=\varphi_{rp}$, longitudinal velocity disappears:

$$u=0,$$

to what it corresponds:

$$\left.\begin{array}{l} F'(\varphi_{rp})=0, \\ Z=z'=0. \end{array}\right\} \qquad (45c)$$

Boundary condition (45b) gives on the axis of the jet:

$$\frac{dz}{d\varphi}\cdot\frac{e^{z}}{\varphi}=1. \qquad (46a)$$

Utilizing equation (46a) as the first approximation for solving stated problem, we obtain:

$$e^{z}=\frac{\varphi^2}{2}-c,$$

but with $\phi=0$:

$$e^{z}=0,$$

therefore

$$c=0;\ e^{z}=\frac{\varphi^{2}}{2};\ z=\ln\frac{\varphi^{2}}{2};\ z'=\frac{2}{\varphi}. \qquad (46b)$$

Similarly how it was done for the slot jet, let us present the solution of differential equation (45) in the form of series/row. The first member of this series/row we will obtain from approximate solution (46b):

$$Z=z'=\frac{2}{\varphi}+\dots$$

Subsequent members of series/row let us find by the method of successive approximations. For this let us first assign the two-termed series/row:

$$Z=z'=\frac{2}{\varphi}+A_{1}\cdot\varphi^{t_{1}} \qquad (47)$$

and let us substitute it in equation (45):

$$-\frac{2}{\varphi^{2}}+A_{1}\cdot t_{1}\cdot\varphi^{t_{1}-1}=\frac{2}{\varphi^{2}}+A_{1}\cdot\varphi^{t_{1}-1}-\frac{4}{\varphi^{2}}-4\cdot A_{1}\cdot\varphi^{t_{1}-1}-A_{1}^{2}\cdot\varphi^{2t_{1}}-$$
$$-\sqrt{\frac{2}{\varphi}+A_{1}\cdot\varphi^{t_{1}}}.$$

Disregarding the members of the second order of smallness ($A_{1}^{2}\cdot\varphi^{2t_{1}}$ and $A_{1}\cdot\varphi^{t_{1}}$) and producing elementary conversions, we obtain:

$$A_{1}\cdot(t_{1}+3)\cdot\varphi^{t_{1}-1}=-\sqrt{2}\cdot\varphi^{-\frac{1}{2}}.$$

Equalizing exponents in the right and left sides of the equality, we find the exponent of the second term of series/row (47):

$$t_{1}=\frac{1}{2}.$$

Comparing coefficients of both of parts of this equality, we compute the coefficient of the second term of series/row (47):

$$A_1 = -\frac{2\cdot\sqrt{2}}{7}.$$

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Thus, the solution of equation (45) in the second approximation/approach appears as follows:

$$z' = Z = \frac{2}{\varphi} - \frac{2\cdot\sqrt{2}}{7}\cdot\varphi. \qquad (48)$$

For the third approximation/approach let us assign the trinomial series/row:

$$Z = z' = \frac{2}{\varphi} - \frac{2\cdot\sqrt{2}}{7}\cdot\varphi + A_2\cdot\varphi^2 \qquad (49)$$

let us also substitute it in initial differential equation (45). After the series/row of the algebraic actions which are completely analogous to those performed above, we compute the third term of the series/row:

$$A_2\cdot\varphi^2 = -\frac{1}{245}\cdot\varphi^2.$$

Continuing calculations in the direction indicated, we can calculate as many as convenient terms of series/row. If, for example, we are restricted by six members, then we will obtain the unknown function in the following form:

$$Z = z' = \frac{2}{\varphi} - 0{,}4\cdot\varphi - 0{,}004\cdot\varphi^2 + 0{,}00082\cdot\varphi^3 + 0{,}00015\cdot\varphi^4 - \\ - 0{,}000014\cdot\varphi^5. \qquad (50)$$

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Integrating equation (50), we find the function:

$$z = \ln \frac{\varphi^2}{2} - 0,27 \cdot \varphi^2 - 0,0013 \cdot \varphi^3 + 0,00018 \cdot \varphi^4 + 0,000025 \cdot \varphi^5 - \\ + 0,000002 \cdot \varphi^6 + \ldots \tag{51}$$

From the equalities:

$$\left.\begin{aligned} F(\varphi) &= e^z, \\ F'(\varphi) &= z' \cdot e^z. \end{aligned}\right\} \tag{52}$$

we compute function F(φ) and its derivative - F'(φ). Finally with the aid of formulas (42) and (43) we determine the relative components of the speed at different points of the cross section of circular source or, which is the same thing, of the basic section of the circular jet:

$$\frac{u}{u_m} = \frac{F'(\varphi)}{\varphi}$$

$$\frac{1}{a} \cdot \frac{v}{u_m} = F'(\varphi) - \frac{F(\varphi)}{\varphi}.$$

Series/rows (50) and (51) are completely suitable in the middle parts of the cross section of jet. However, in the region of the boundary of the jet - near $\varphi = \varphi_{rp}$ - these series/rows possess bad convergence. Therefore for the boundary layers of jet it is necessary to obtain another solution of differential equation (45). The new solution can be obtained from boundary condition (45c), according to which on the jet boundary $(\varphi = \varphi_{rp})$ disappears function Z:

$$Z = z'(\varphi_{rp}) = 0.$$

In fact, if

$$z' = Z \to 0,$$

then in equation (45):

$$Z' = -\sqrt{Z}$$

and

$$2\cdot\sqrt{Z} = -\varphi + c.$$

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When $\varphi = \varphi_{гр}$:

$$Z = 0,$$

therefore

$$c = \varphi_{гр}.$$

Thus, in the first approximaticn,:

$$Z = \frac{1}{4}\cdot(\varphi_{гр} - \varphi)^2. \qquad (53)$$

Converting/transferring to the second approximation/approach, let us present solution in the form cf the binomial:

$$Z = \frac{1}{4}\cdot(\varphi_{гр} - \varphi)^2 + B_1\cdot(\varphi_{гр} - \varphi)^3.$$

After the substitution of this binomial in equation (45), will be determined by already known methcd its second term:

$$B_1\cdot(\varphi_{гр} - \varphi)^3 = -\frac{1}{8\cdot\varphi_{гр}}\cdot(\varphi_{гр} - \varphi)^3.$$

Continuing calculations in the same direction, it is possible to find any number of terms of series/rcw. If we are restricted by five members of series/row, then we will obtain:

$$z' = Z = \frac{1}{4}\cdot(\varphi_{\text{rp}} - \varphi)^2 - \frac{1}{8\cdot\varphi_{\text{rp}}}\cdot(\varphi_{\text{rp}} - \varphi)^3 - \frac{3}{64\cdot\varphi_{\text{rp}}^2}\cdot(\varphi_{\text{rp}} - \varphi)^4 + \left(\frac{1}{64} - \frac{3}{128\cdot\varphi_{\text{rp}}^3}\right)\cdot(\varphi_{\text{rp}} - \varphi)^5 - \left(\frac{19}{1\,280\cdot\varphi_{\text{rp}}} + \frac{133}{10\,240\cdot\varphi_{\text{rp}}^4}\right)\cdot(\varphi_{\text{rp}} - \varphi)^6. \quad (54)$$

Equation (54) gives a good convergence near the jet boundary, i.e., exactly in that region, for which is unsuitable equation (50).

Therefore during the calculation of functions z, $F(\varphi)$ and $F'(\varphi)$ in the middle layers jets use equation (50), while in the boundary layers is utilized equation (54). In order to determine the value of the relative ordinate of jet boundary (φ_{rp}), let us compare in any known to point jet (for example, at point $\varphi=2$) of equation (50) and (54):

$$Z_{50} = Z_{54}. \quad (55)$$

By producing then calculations with the method of successive approximations, let us find:

$$\varphi_{\text{rp}} = 3{,}4. \quad (56)$$

It must be noted that condition (56) gives the possibility to replace with numbers all coefficients of series/row (54):

$$Z = 0{,}25\cdot(3{,}4-\varphi)^2 - 0{,}037\cdot(3{,}4-\varphi)^3 - 0{,}004\cdot(3{,}4-\varphi)^4 + 0{,}015\cdot(3{,}4-\varphi)^5 - 0{,}0173\cdot(3{,}4-\varphi)^6. \quad (57)$$

Now, when all basic functions of the turbulent source of circular jet are determined, we will use by them and let us compose tables 2 and Fig. 12-13 longitudinal and transversing speeds of the cross section of the basic section of circular jet.

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In table 2 and Fig. 12-13 speeds u and v are measured in the portions of axial velocity (u_m). The latter can be found from the expression:

$$u_m = \frac{m}{a \cdot x}^{1)}. \qquad (58)$$

FOOTNOTE [1]. See equality (42). ENDFOOTNOTE.

We will use the constancy of the momentum of circular jet in order to give to equality (58) more convenient form.

We have:

$$2 \cdot \pi \cdot \rho \cdot \int_0^{y_{rp}} u^2 \cdot y \cdot dy = \pi \cdot \rho \cdot u_0^2 \cdot R_0^2 = \text{const}.$$

Here y_{rp} - absolute ordinate of jet boundary; ρ - air density; R_0 - radius of initial jet cross-sectional area; u_0 - discharge velocity.

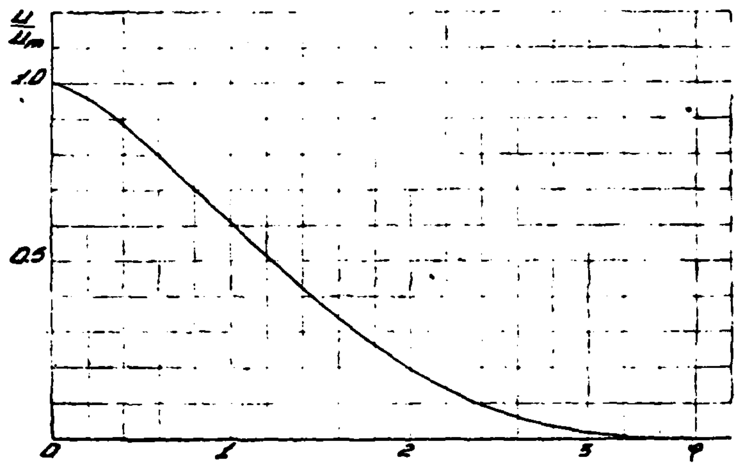

Fig. 12. Distribution of the longitudinal velocities in the section of circular jet.

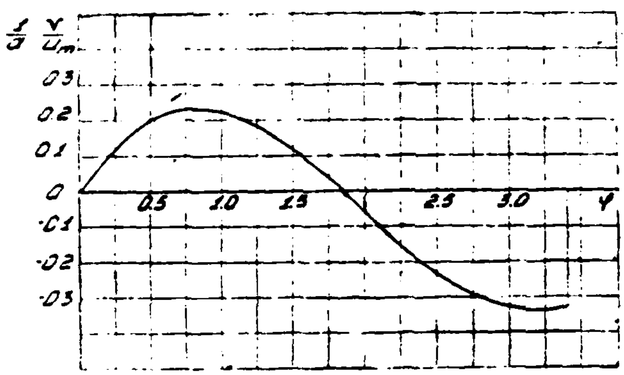

Fig. 13. Distribution of transversing speeds in section of circular jet.

Table 2.

φ	$\frac{u}{u_m} = \frac{F'(\varphi)}{\varphi}$	$\frac{1}{a} \cdot \frac{V}{u_m}$
0	1	0
0,1	0,984	0,050
0,2	0,958	0,100
0,3	0,922	0,144
0,4	0,885	0,178
0,5	0,843	0,200
0,6	0,795	0,220
0,7	0,748	0,230
0,8	0,700	0,240
0,9	0,653	0,233
1,0	0,606	0,225
1,1	0,555	0,210
1,2	0,510	0,190
1,3	0,470	0,170
1,4	0,425	0,140
1,5	0,378	0,110
1,6	0,340	0,080
1,7	0,300	0,040
1,8	0,265	0
1,9	0,230	−0,033
2,0	0,198	−0,066
2,1	0,169	−0,100
2,2	0,140	−0,140
2,3	0,117	−0,180
2,4	0,094	−0,219
2,5	0,075	−0,237
2,6	0,059	−0,270
2,7	0,046	−0,295
2,8	0,034	−0,310
2,9	0,024	−0,323
3,0	0,017	−0,334
3,1	0,011	−0,340
3,2	0,007	−0,345
3,3	0,003	−0,340
3,4	0	−0,335

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After the replacement of variable/alternating, we obtain:

$$\frac{a^2 \cdot x^2}{R_0^2} \cdot \frac{u_m^2}{u_0^2} \int_0^{\varphi_{rp}=3.4} \frac{u^2}{u_m^2} \cdot \varphi \cdot d\varphi = 0.5$$

or otherwise

$$\frac{u_m}{u_0} = \frac{0{,}707}{\frac{a \cdot x}{R_0} \cdot \sqrt{\int_0^{3,4} \frac{[F'(\xi)]^2}{\xi} d\xi}}$$

The integral, which stands under the root is equal to:

$$\int_0^{3,4} \frac{F'(\xi)^2}{\xi} \cdot d\xi = 0{,}536\ ^{1}),$$

which gives the possibility tc obtain the final formula of the axial velocity of the basic secticn of the circular jet:

$$\frac{u_m}{u_0} = \frac{0{,}96}{\frac{a \cdot x}{R_0}}. \qquad (59)$$

FOOTNOTE [1]. This integral we calculated according to the method of trapezoids with the aid of Table 2. ENDFOOTNOTE.

In transient jet cross-sectional area, from which begins basic section, axial velocity is equal to discharge velocity:

$$\frac{u_m}{u_0} = 1.$$

Hence we find the relative abscissa of the transition section of the circular jet:

$$\frac{a \cdot L_0}{R_0} = 0{,}96. \qquad (60)$$

All obtained functions of the turbulent source of circular jet are suitable only in the region of the basic section where

$$x \geqslant L_0.$$

§5. Initial section of free jet.

The experiences of the Goettingen wind-tunnel laboratory and TsAGI show that in the region of the first two bores of circular jet the high-speed/high-velocity profile/airfoil of its boundary layer completely coincides with the high-speed/high-velocity profile/airfoil of the free boundary of infinite plane flow[2].

FOOTNOTE [2]. G. N. Abramovich. Aerodynamics of flow in open wind-tunnel test section. Pt. I, page 23-24. Transactions of TsAGI iss. 223. ENDFOOTNOTE.

At the same time, at the end of the initial section - in the transient section - the velocity profile must be the same as in the basic section of jet. In other words, for the elongation/extent of initial section must be observed the continuous conversion of velocity fields.

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Let us give the series/row of the experimental diagrams of Goettingen institute[1], which clearly illustrate this position:

FOOTNOTE [1]. Ergebnisse der Aerodynamische Versuchsanstalt zu Goettingen. II Lieferung. 1923. ENDFOOTNOTE.

1) Fig. 14 depicts the field of velocity heads of free jet at a distance of 1.5 m from the nozzle. Diameter of nozzle D_0=0.137 m. Discharge velocity u'_0=40 m/s;

2) Fig. 15 - the same, at a distance of 1.0 m;

3) Fig. 16 - the same, at a distance of 0.5 m;

4) Fig. 17 - the same, at a distance of 0.25 m;

5) Fig. 18 - the same, at a distance of 0.01 m.

The given diagrams are written by the pressure recorder of Bartels's system. Next to the zigzag empirical curves are plotted/applied the theoretical profiles, borrowed from the theory of the basic section of jet. Of course in the initial section (Fig. 16 and 17) these theoretical profiles/airfoils are constructed only in the region of boundary layer.

As we see, within the limits of basic section (Fig. 14 and 15) the experimental and theoretical velocity profiles virtually coincide, while in the boundary layer of initial section (Fig. 16 and 17) the velocity field is deformed and differs from theoretical all the more than the selected section it is nearer to the nozzle. In order to show that near the nozzle the high-speed/high-velocity profile/airfoil of the jet of the same, as in infinite plane flow, let us turn to the measurements of velocity heads into the wind tunnel of Goettingen institute. These measurements are produced in the boundary layer of the open test section of Prandtl's large duct (diameter D_0=2.25 m) at a distance of 1.12 m from the nozzle.

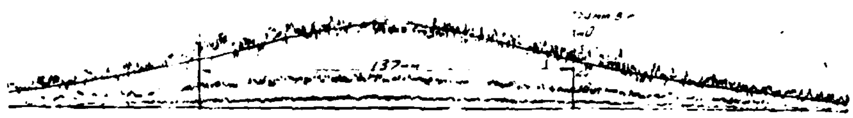

Fig. 14. Fields of velocity heads of free jet at a distance of 1.5 m from the nozzle with a diameter of 0.137 m.

Fig. 15. Fields of velocity heads of free jet at a distance of 1.0 m from nozzle with a diameter of 0.137 m.

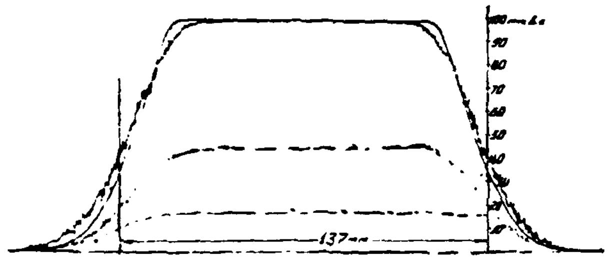

Fig. 16. Fields of velocity heads of free jet at a distance of 0.01 m from nozzle with a diameter of 0.137 m.

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Experimental curve is constructed next to the theoretical curve of plane flow and completely it ccincides with it (Fig. 19). Thus, it is possible to consider it established/installed that in the boundary layer of the initial section cf free jet occurs the gradual transition from the velocity field of infinite plane flow to the velocity field of the basic section of jet.

Analyzing the position indicated, it is possible to make the following conclusions:

1. During the study of the first two bores of jet should be utilized laws of the free bourdary of plane flow. So this and is made during the development of the aerodynamic design of open wind-tunnel test section.

2. During study of initial section as parts of entire jet, it is possible to disregard strain cf velocity fields and to consider that in all sections of boundary layer of initial section velocity fields are similar to those which are established in basic section of jet. This conclusion/output will become especially convincing, if we focus attention on the fact that the deviation from it is made by noticeable only in that region of the jet where the boundary layer thickness is small and its "specific gravity/weight" in the total balance of masses and energies is insignificant.

Subsequently, relying on conclusion/output (2), we will apply one and the same velocity profile both for the basic section of free jet and for the boundary layer of its initial section.

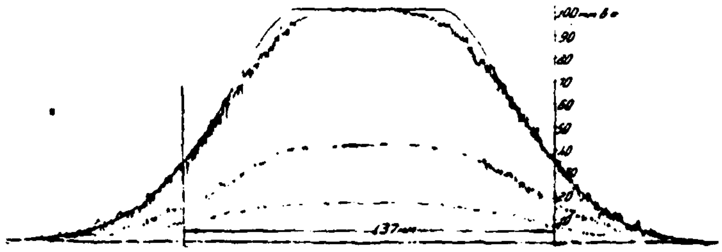

Fig. 17. Fields of velocity heads of free jet at a distance of 0.25 m from the nozzle with a diameter of 0.137 m.

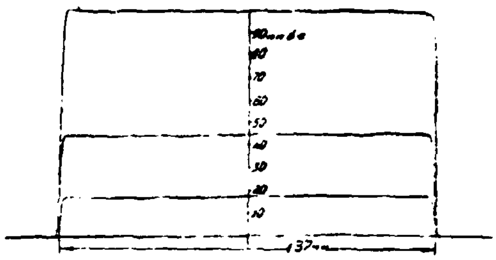

Fig. 18. Fields of velocity heads of free jet at a distance of 0.5 m from nozzle with a diameter of 0.137 m.

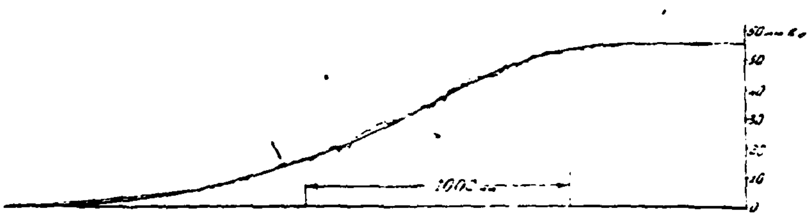

Fig. 19. Fields of velocity heads in boundary layer of free jet at a distance of 1.12 m from nozzle with a diameter of 2.25 m.

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Repeatedly it was mentioned, that is investigated this case of the free jet when in the initial section occurs the uniform field of velocities[1].

FOOTNOTE [1]. Correction for the nonuniformity of the initial field of velocities will be introduced below. ENDFOOTNOTE.

In this case, the initial boundary layer thickness of jet is equal to zero and it is possible to consider that it begins its existence at nozzle discharge edge (Fig. 20).

Let us use to the boundary layer of the initial section of jet the basic hypothesis of Prandtl's free turbulence [see equality (2)]:

$$l' = c' \cdot S^{2)}. \quad (61)$$

FOOTNOTE [2]. l' - mixing length in this section of boundary layer; c' - constant, different from constant c of basic section; S - distance of this section from the edge of nozzle. ENDFOOTNOTE.

Combining this hypothesis with the assumption about the similarity of velocities, we come (as in §2) to the conclusion about the straightness of the external and internal boundaries of the boundary layer of the initial section of jet. Furthermore, any ray/beam C, which rises from from the edge of nozzle and lying within the limits of boundary layer, must be the line of equal velocities.

These facts attest to the fact that during the study of initial section to rationally arrange the origin of coordinates at the edge of nozzle. Axis Y direct inside the flow. As the coordinates let us select S and $\phi'=Y/a' \cdot S$, where

$$a' = \sqrt[3]{2(c')^2}.$$

In this coordinate system internal and external boundaries of the boundary layer of initial section are respectively determined by relative ordinates ϕ'_1 and ϕ'_2.

Utilizing Fig. 21, let us find the basic shape factors of the initial section of jet.

From the similarity of triangles we have:

$$\frac{h_0}{b_0} = \frac{h_0}{b_n} = \frac{1}{a \cdot \varphi_{\text{гр}}}. \quad (62)$$

Hence relative distance from the pole of the jet to its initial section - the depth of the pole:

$$\frac{a \cdot h_0}{b_0} = \frac{1}{\varphi_{\text{гр}}}. \quad (63)$$

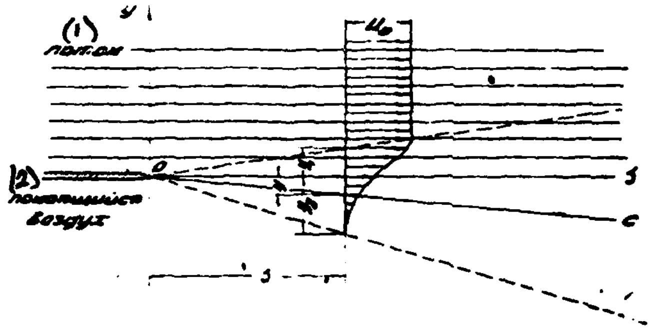

Fig. 20. Boundary layer in the initial section of free jet.

Key: (1). Then. (2). Quiescent air.

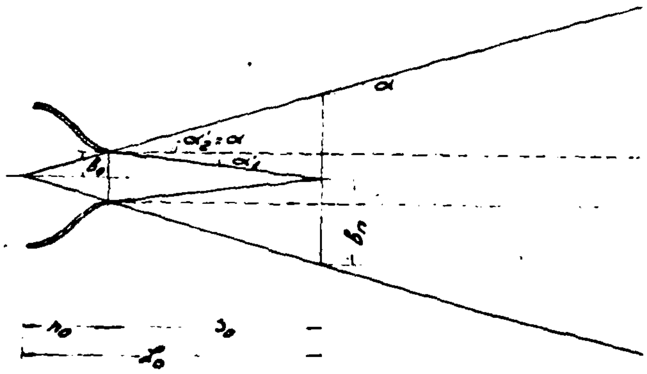

Fig. 21. Geometric diagram of free jet.

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The length of initial section (S_0) is equal to the difference between the polar distance of transient section and with a depth of the pole of:

$$\frac{aS_0}{b_0} = \frac{a \cdot L_0}{b_0} - \frac{a \cdot h_0}{b_0}. \quad (64)$$

Mixing length at the end of the initial section is equal to mixing length in the beginning of the basic section:

$$l_0' = l_0,$$

whence

$$\frac{c'}{c} = \frac{L_0}{S_0}, \quad (65)$$

but

$$a' = \sqrt[3]{2(c')^2}$$

and

$$a = \sqrt[3]{2(c)^2},$$

therefore

$$\frac{a'}{a} = \sqrt[3]{\frac{L_0^2}{S_0^2}}. \quad (66)$$

The region of constant velocities ($u=u_0$) toward the end of the initial section is eliminated, thanks to which:

$$\operatorname{tg} \alpha_1' = a \cdot \varphi_1' = \frac{b_0}{S_0};$$

in other words, the relative ordinate of the internal boundary of the boundary layer:

$$\varphi_1' = \frac{1}{\frac{a'}{a} \cdot a \cdot \frac{S_0}{b_0}} \quad (67)$$

Boundary layer thickness at the end of the initial section is equal to the half-width of the transient section:

$$b_n = a' \cdot (\varphi_1' - \varphi_2') \cdot S_0 = a \cdot \varphi_{rp} \cdot L_0^{1)},$$

whence

$$\varphi_1' - \varphi_2' = \varphi_{rp} \cdot \sqrt{\frac{L_0}{S_0}} \cdot$$

FOOTNOTE [1]. Here ϕ'_2 is undertaken with the negative sign, because y axis is directed inside the flcw. ENDFOOTNOTE.

Thus, the ordinate of boundary layer edge in the initial section of the jet:

$$\varphi_2' = \varphi_1' - \varphi_{rp} \cdot \sqrt[3]{\frac{L_0}{S_0}} \cdot \qquad (68)$$

The obtained general formulas of the geometric parameters of jet are suitable both for the circular and for the slot jet. In the use/application to each of the jet individually, all these formulas are replaced by numbers.

Let us examine the basic properties of the initial section of slot jet.

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In §3 was found the polar distance of the transient section:

$$\frac{a \cdot L_0}{b_0} = 1,44$$

and the relative ordinate of the boundary of the basic section of the slot jet:

$$\varphi_{rp} = 2,412.$$

Hence the depth of the pole of the slot jet:

$$\frac{a \cdot h_0}{b_0} = 0,41,$$

the length of the initial section:

$$\frac{a \cdot S_0}{b_0} = 1,03,$$

the value of the empirical coefficient:

$$a' = 1,25 \cdot a,$$

the relative ordinate of the internal boundary of the boundary layer:

$$\varphi_1' = 0,77$$

and the relative ordinate of boundary layer edge:

$$\varphi_2' = -1,93.$$

The velocity field of the boundary layer of initial section, as it was accepted, similar the velocity field of the basic section of jet. Therefore in order to obtain the field of the boundary layer of slot jet, it suffices Tables 1 and Fig. 9 to change in accordance with the ordinates of boundary layer, after accepting ϕ'_1=0.77 instead of ϕ_1=0 and ϕ'_2=-1.93 instead of $\varphi_{rp} = 2,4$.

As a result of this rearrangement are obtained Table 3[1] and Fig. 22.

FOOTNOTE [1]. Table 3 gives values u/u_0 instead of $\frac{u}{u_m}$ since in initial section $u_m = u_0$ ENDFOOTNOTE.

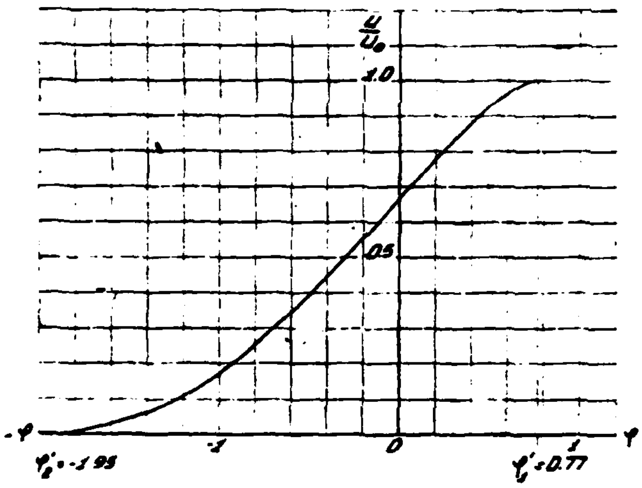

Fig. 22. The velocity field in the boundary layer of the initial section of slot jet.

Tabl 3.

φ'	0,77	0,7	0,6	0,5	0,4	0,3	0,2	0,1	0
$\frac{u}{u_0}$	1	0,991	0,960	0,920	0,880	0,830	0,777	0,721	0,670

φ'	−0,1	−0,2	−0,3	−0,4	−0,5	−0,6	−0,7	−0,8
$\frac{u}{u_0}$	0,615	0,560	0,503	0,447	0,392	0,340	0,292	0,250

φ'	−0,9	−1,0	−1,1	−1,2	−1,3	−1,4	−1,5
$\frac{u}{u_0}$	0,210	0,173	0,140	0,110	0,085	0,061	0,043

φ'	−1,6	−1,7	−1,8	−1,9	−1,93
$\frac{u}{u_0}$	0,029	0,018	0,009	0,001	0

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For calculating the geometric parameters of the initial section of circular jet it is possible to use the fact that the polar distance of transient section is equal to:

$$\frac{a \cdot L_0}{R_0} = 0{,}96,$$

while the relative ordinate of the boundary of basic section has a value:

$$\varphi_{rp} = 3{,}4.$$

Hence we find the relative depth of the pole:

$$\frac{ah_0}{R_0}=0{,}29,$$

the relative length of the initial section:

$$\frac{a\cdot S_0}{R_0}=0{,}67,$$

the coefficient of flow pattern in the initial section:

$$a'=1{,}28\cdot a,$$

the relative ordinate of the internal boundary of the boundary layer:

$$\varphi_1'=1{,}17,$$

and finally the relative ordinate of the outer edge of the initial section of the circular jet:

$$\varphi_2'=-2{,}67.$$

In order to complete the analysis of the initial section of circular jet, let us compose Fig. 23 and Table 4 of relative velocities in the boundary layer of initial section. Table 4 and graph/curve by 23 are constructed with the aid of Table 2. In this case the boundary ordinates of the basic section of the circular jet (ϕ_1=0 and $\varphi_{rp}=3{,}4$) are replaced by the boundary ordinates of the boundary layer of initial section (ϕ'_1=1.17 and ϕ'_2=2.67).

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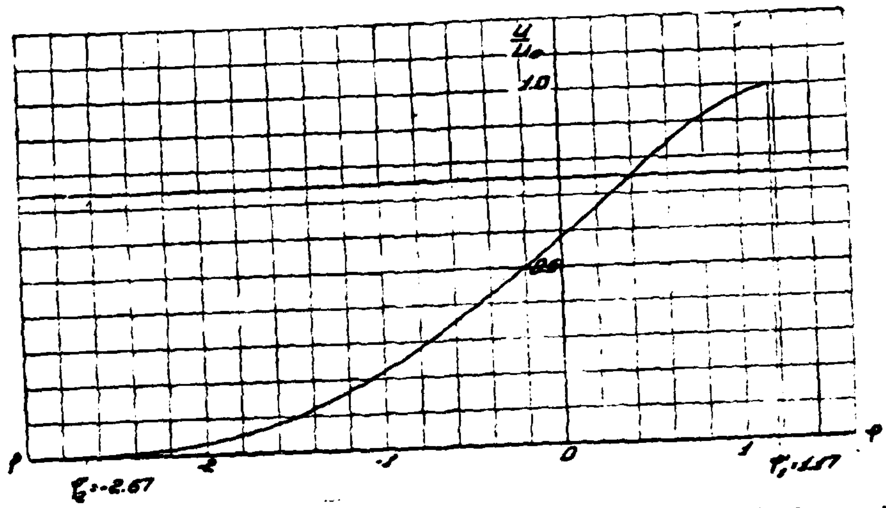

Fig. 23. Velocity field in boundary layer of initial section of circular jet.

Table 4.

φ	1,17	1,1	1,0	0,9	0,8	0,7	0,6	0,5
$\frac{u}{u_0}$	1	0.990	0,971	0,945	0,914	0,879	0,838	0,795
φ	0,4	0,3	0,2	0,1	0	−0.1	−0.2	−0,3
$\frac{u}{u_0}$	0,751	0,7[illegible]0	0,668	0,625	0,582	0.540	0,500	0,464
φ	−0,4	−0.5	−0,6	−0,7	−0,8	−0.9		
$\frac{u}{u_0}$	0,428	0,389	0,350	0,316	0,280	0,250		
φ	−1,0	−1,1	−1,2	−1,3	−1,4	−1,5	−1,6	−1.7
$\frac{u}{u_0}$	0,218	0,188	0.161	0.140	0.120	0,100	0,081	0,06[illegible]
φ	−1,8	−1,9	−2,0	−2,1	−2,2	−2,3	−2,4	−2,5
$\frac{u}{u_0}$	0,051	0,041	0,032	0,025	0,019	0,012	0,010	0,005
φ	−2,6	−2,67						
$\frac{u}{u_0}$	0,001	0						

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§6. Basic conclusion/output of free boundaries theory and their experimental check.

A) Circular jet.

The construction of the configurations of circular free jet is conducted as follows (Fig. 24):

1) Find the pole of jet whose depth[1]:

$$\frac{h_0}{R_0} = \frac{0{,}29}{a} \qquad (69)$$

and conduct through it and through nozzle discharge edge the rays/beams of external jet boundary. Tangent of the divergence angle of the outer edge:

$$\operatorname{tg} \alpha = a' \cdot \varphi_2' = a \cdot \varphi_{\text{гр}} = 3{,}4 \cdot a. \qquad (70)$$

FOOTNOTE [1]. On the selection of value a it will be said below. ENDFOOTNOTE.

2. They find location of transient jet cross-sectional area:

$$\frac{S_\bullet}{R_0} = \frac{0.67}{a} \qquad (71)$$

and are constructed it. It is interesting to note that a radius of transient section is a constant value and does not depend on jet structure:

$$\frac{R_*}{R_\bullet} = \frac{L_0}{h_0} = 3{,}3. \qquad (72)$$

3. Connect center of transient section with edge nozzles and they obtain thus, boundary of nucleus of constant velocities ($u=u_0$).

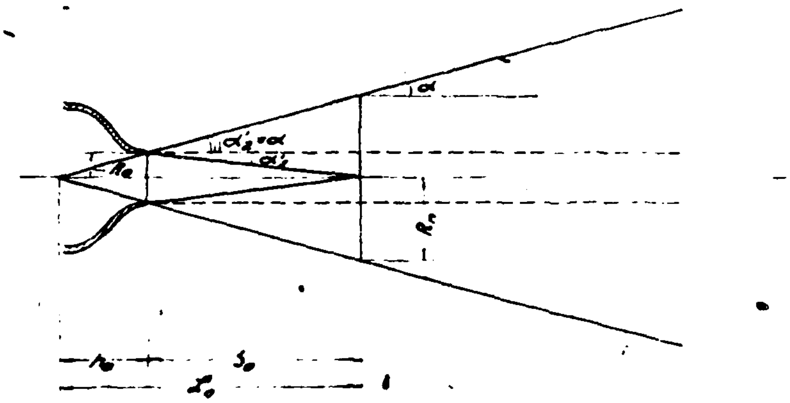

Fig. 24. Geometric diagram of free circular jet.

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The tangent of the convergence angle of the boundary of the nucleus of constant velocities is equal to:

$$\operatorname{tg} \alpha_1' = a' \cdot \varphi_1' = 1{,}28 \cdot a \cdot \varphi_1' = 1{,}5 \cdot a. \tag{73}$$

The divergence angle of the boundary layer of the initial section of circular jet is comprised from the sum of angles α'_1 and $\alpha'_2=\alpha$:

$$\beta = \alpha_1' + \alpha_2' = \operatorname{arctg}(1{,}5 \cdot a) + \operatorname{arctg}(3{,}4 \cdot a). \tag{74}$$

The width of boundary layer in the arbitrary section of initial section comprises:

$$\frac{b_{\text{п. с}}}{R_0} = \frac{a' \cdot (\varphi_1' - \varphi_2') \cdot S}{R_0} = 4{,}9 \cdot a \frac{S}{R_0}. \tag{75}$$

A finally complete radius of jet at the assigned distance (S) from the nozzle is measured by the value:

$$\frac{R_{\text{гр}}}{R_0} = \frac{S + h_0}{h_0} = 3{,}4 \cdot \frac{aS}{R_0} + 1. \tag{76}$$

The only experimental coefficient of theory (coefficient a) depends on jet structure.

The analysis of experiments of Trupel, Zimm, Gettingen aerodynamic institute, Turkus and Syrkin [1] shows that in the very broad band of Reynolds numbers (from 20000 to 4000000) coefficient a does not depend on Reynolds number.

FOOTNOTE [1]. See bibliographical directory at the end of the article. ENDFOOTNOTE.

At the same time the value of quantity a somewhat changes with a change in the velocity profile in the beginning of jet. With this a it increases with an increase in the nonuniformity of high-speed/high-velocity profile/airfoil.

After working/treatment of experiments indicated it was possible to construct the graph/diagram of 25 dependences of coefficient a of circular jet on the elongation of its initial field of velocities $\left(\frac{u_{max\,0}}{u_{cp\,0}}\right)$.

From Fig. 25 it is evident that for the completely uniform field of velocities - a=0.066. In the series/row with this for the completely steady turbulent field of velocities $\left(\frac{u_{max}}{u_{cp}} \cong 1{,}25\right) - a = 0{,}076.$

Fig. 25 relates to the "natural" turbulent jets. With the aid of the artificial agitation of flow it is possible to sharply increase coefficient of a which in turn, will lead to the more rapid fading of jet.

The artificial agitation of free jet was conducted by eng. Syrkin, who established/installed in initial jet cross-sectional area (at the output from the duct) turbulence generating grid and inclined at angle of 45° toward the axis the guides of blade. Turbulence generating grid raised coefficient of a to value of a=0.089. However, guide apparatus caused a=0.27.

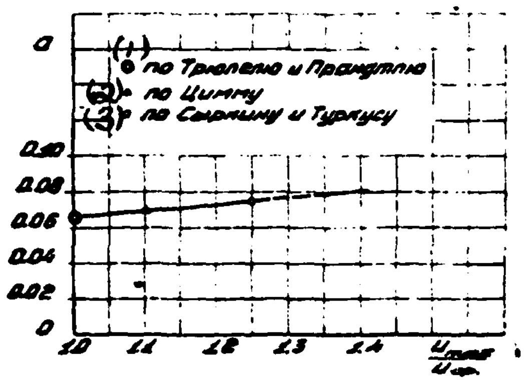

Fig. 25. Dependence of the coefficient of jet structure a on the initial velocity field of circular jet.

Key: (1). according to Trupel and Prandtl. (2). according to Zimm. (3). according to Syrkin and Turkus.

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The velocity on the axis of the basic section of circular jet, measured in the portions of the discharge velocity, is determined by the formula [see equality (59)]:

$$\frac{u_m}{u_0}=\frac{0{,}96}{\frac{a\cdot x}{R_0}}=\frac{0.96}{\frac{a\cdot S}{R_0}+0{,}29}. \qquad (77)$$

In the limits of the initial section where

$$\frac{a\cdot S}{R_0}\leq 0{,}67,$$

axial velocity is constant/invariable and equal tc discharge velocity:

$$\frac{u_m}{u_0} = 1 \qquad (78)$$

Fig. 26 connects together fcrmulas (77) and (78). On it is depicted the curve of a change in the axial velocity along the entire free jet. In Fig. 26 are plctted the experimental points of Trupel, Zimm, Gettingen institute, Turkus and Syrkin [1].

FOOTNOTE [1]. See bibliographical directory at the end of the article. ENDFOOTNOTE.

In this case in accordance with graph/curve by 25, are accepted the following values of constants a:

1) in experiments of Trupel and Gettingen experiments (initial velocity field uniform) ... a=0.066.

2) in the experiences cf Zimm $\left(\frac{u_{max}}{u_{cp}} = 1,10\right)$ ... a=0.070.

3) in experiments of Turkus and Syrkin $\left(\frac{u_{max}}{u_{cp}} = 1,25\right)$ ... a=0.076.

Fig. 26 testifies about the excellent experimental confirmation of the character of thecretical curve of axial velocities along the circular jet.

The theoretical velocity distribution law in the cross section of circular jet was above compared with Gettingen experiments (see Fig. 14, 15, 16, 17, 18 and 19), which it completely confirmed. In addition to this is given the comparison of the law cf the cross field of the velocities [see expression (42)] circular jet with experiments of Trupel (Fig. 27). Experimental points Fig. 27 are borrowed from Fig. 4. Thecretical curve is constructed in the form:

$$\frac{u}{u_m} = f_1\left(\frac{\varphi}{\varphi_{\frac{u_m}{2}}}\right)$$

instead of

$$\frac{u}{u_m} = f(\varphi) = \frac{F'(\varphi)}{\varphi}. \tag{7[illegible]}$$

As we see, and here the thecry cf circular jet coincides with experiment.

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B). Slot jet.

The construction of the ccnfigurations of slot jet is conducted by accurately the same methcd, as in the case of circular jet, with the only difference that the are utilized cther numerical values of geometric parameters. Depth cf the pole of the slot jet:

$$\frac{h_0}{b_0}=\frac{0{,}41}{a}. \tag{80}$$

Tangent of the divergence angle of the outer edge of the slot jet:

$$\operatorname{tg}\alpha=a'\cdot\varphi_2'=a\cdot\varphi_{\text{гр}}=2{,}4\cdot a. \tag{81}$$

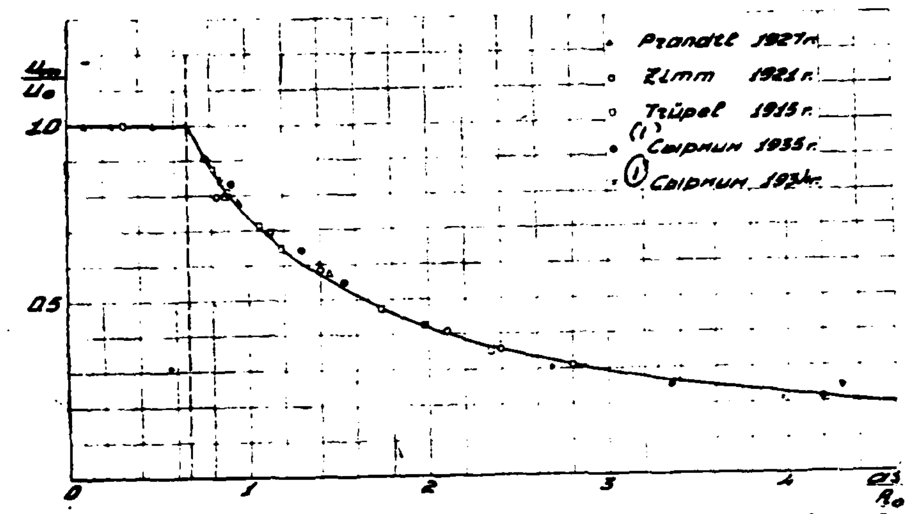

Fig. 26. Velocity change along the axis of circular jet.

Key: (1). Syrkin.

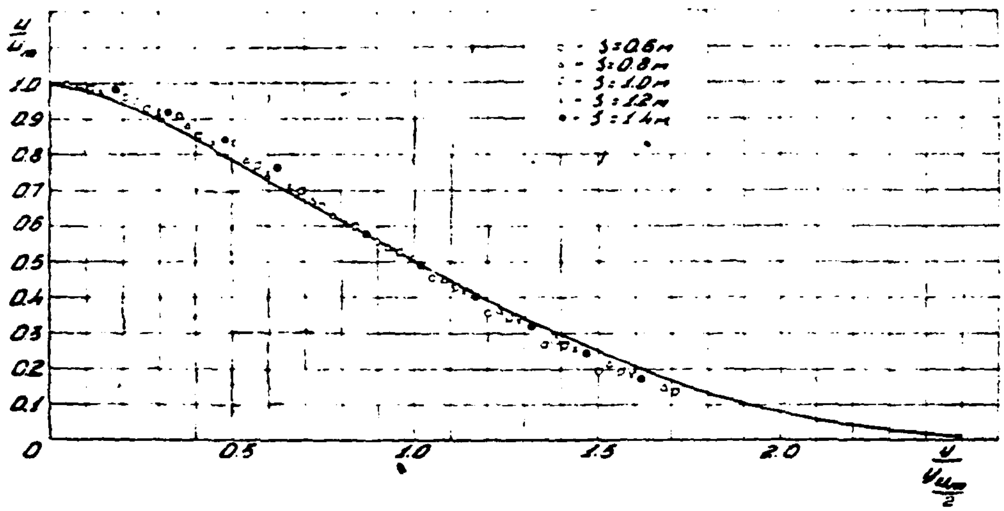

Fig. 27. Comparison of the theoretical distribution of the velocities in the cross section of circular jet with experiments of Trupel.

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Length of the initial section:

$$\frac{S_n}{h_n} = \frac{1,03}{a} \quad (82)$$

Width of the transient section:

$$\frac{b_n}{h_n} = 3,5 \quad (83)$$

Tangent of the convergence angle of the boundary of the nucleus of the constant velocities:

$$\operatorname{tg} \alpha_1' = a' \cdot \varphi_1' = 1,25 \cdot a \cdot \varphi_1 = 0,96 \cdot a. \quad (84)$$

Divergence angle of the boundary layer of the initial section:

$$\beta = \alpha_1 + \alpha_2 = \operatorname{arctg}(0{,}96 \cdot a) + \operatorname{arctg}(2{,}4 \cdot a). \quad (85)$$

Width of boundary layer in the arbitrary section of the initial section:

$$\frac{b_{\text{п.с}}}{b_0} = \frac{a' \cdot (\varphi_1' - \varphi_2') \cdot S}{R_0} = 3{,}3 \cdot a \cdot \frac{S}{R_0}. \quad (86)$$

Complete width of the slot jet:

$$\frac{b_{\text{гр}}}{b_0} = \frac{S + h_0}{h_0} = 2{,}4 \cdot a \cdot \frac{S}{b_0} + 1. \quad (87)$$

For the establishment of the values of coefficient a of slot jet we do not have available so vast experimental material as in the case of circular jet. Therefore it is impossible to establish/install dependence of a on $\frac{u_{max}}{u_{cp}}$ for the slot jet. Nevertheless, by analogy with circular jet, it is possible to be confident in the fact that the coefficient and of the slot jet virtually not of Reynolds's hall, but it grows/rises by increase of degree of irregularity of initial velocity field and with an increase in the initial turbulence of jet. From experiments of Portmann, Proskura and Turkus we found that values a for the slot jet with "natural" turbulence oscillate in the limits:

$$a = 0{,}09 - 0{,}12.$$

In this case the smaller values correspond to the more uniform initial velocity fields.

Velocity for the axis cf the basic section of slot jet is expressed by the formula [see eqcality (32)].

$$\frac{u_m}{u_0}=\frac{1.2}{\sqrt{\frac{a\cdot x}{b_0}+0.41}}=\frac{1.2}{\sqrt{\frac{aS}{b_0}+0.41}}. \qquad (88)$$

In the limits of the initial section where

$$\frac{aS}{b_0}<1.03,$$

axial velocity is constant:

$$\frac{u_m}{u_0}=1. \qquad (89)$$

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In Fig. 28 is plotted the curve of the axial velocities in the initial and basic secticns cf slct jet. Fcr ccmparison of theory with experiment Fig. 28 gives the experimental pcints cf Fortmann, Turkus and Proskura.

Moreover:

for the pcints of Fortmarn is accepted ... a=0.11.

for the pcints of Turkus is accepted ... a=0.09.

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ON THE THEORY OF A FREE AIR JET AND ITS APPLICATION,(U)

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for the points of Proskura is accepted ... a=0.12.

Fig. 28 attests to the fact that the character of theoretical curve of axial velocities along the slot jet is confirmed by experiments.

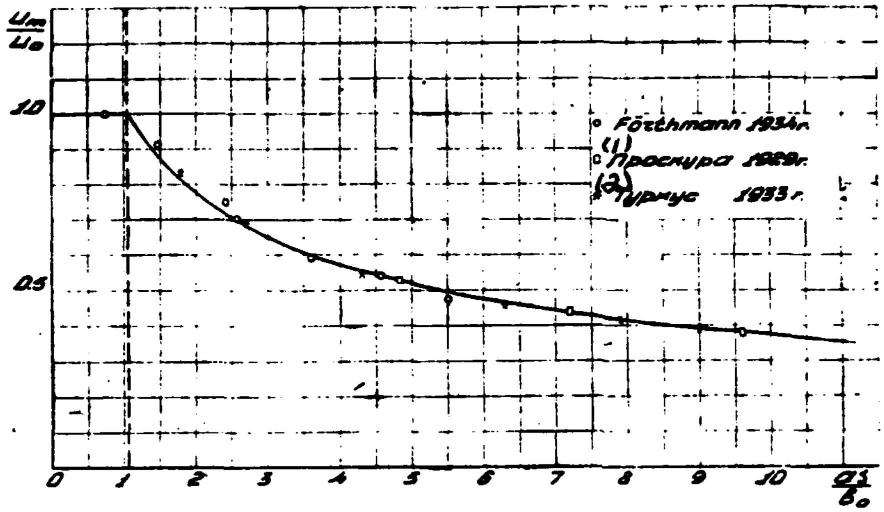

Fig. 28. Velocity change along the axis of slot jet.

Key: (1). Proskura. (2). Turkus.

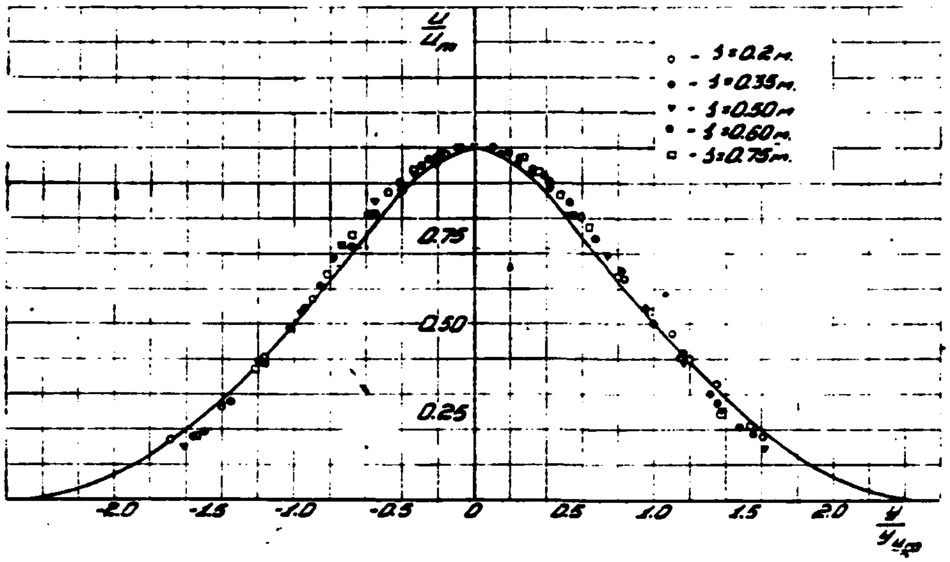

Fig. 29. Comparison of the theoretical distribution of the velocities in the cross section of slot jet with experiments of Fortmann.

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The theoretical profile of the velocities in the cross section of slot jet is also confirmed by experiment, of what it is not difficult to be convinced from Fig. 29. In Fig. 29 are plotted experimental the points of Fortmann which are rebuilt here from Fig. 7. Is here depicted the theoretical curve:

$$\frac{u}{u_m}=f\left(\frac{\varphi}{\varphi_{\frac{u_m}{2}}}\right),$$

constructed with the aid of Tables 2.

§ 7. On the diffusion of heat and gas admixtures/impurities in the free jet.

In the engineering practice frequently it is necessary to deal concerning the free jets which are contaminated by gas admixtures/impurities and have a temperature, different from that surrounding.

The solution of the problem abcut the propagation of gas admixtures/impurities and heat from the quiescent air into the jet (and vice versa) only possible after will become known the laws of a change in the temperatures and gas concentrations along the jet and in its cross sections.

Experiments of Syrkin show that the temperature fields of free jet they are similar to its velocity fields [1].

FOOTNOTE [1]. In more detail this question examines the article of Ruden (Ruden) see reference indicator. ENDFOOTNOTE.

Excellently illustrates this phenomenon Fig. 30, on which is depicted the empirical curve of axial velocity and are plotted/applied the experimental points, which reflect a change in the temperature along the axis of jet. It must be noted that the temperature points express the ratio of the excess temperature in this place to the excess temperature in the beginning of the jet:

$$\frac{T - T_{ном}}{T_0 - T_{ном}} = \frac{\Delta T}{\Delta T_0}.$$

Here T - temperature in this place of jet;

T_0 - temperature in the beginning of jet;

$T_{ном}$ - temperature of location.

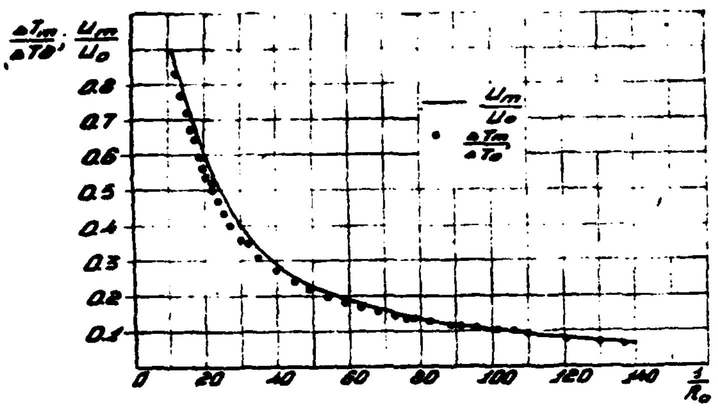

Fig. 30. Comparison of the laws cf a change in the velocity and excess temperature along the axis of circular jet according to experiments of Syrkin.

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The mathematical expression of the similarity of temperature and high-speed/high-velocity fields on the axis of jet appears as follows:

$$\frac{\Delta T_m}{\Delta T_0} = \frac{u_m}{u_0}. \quad (90)$$

The same condition in the cross section of jet takes the following form:

$$\frac{\Delta T}{\Delta T_m} = \frac{u}{u_m}. \quad (91)$$

One should assume that the velocity fields are similar not only temperature fields, but to the equal degree and the fields of the concentrations:

$$\frac{\Delta u}{u_0}=\frac{\Delta T}{\Delta T_0}=\frac{\Delta g}{\Delta g_0}. \quad (92)$$

Here Δg - concentration difference in this place of jet and outside it.

Δg_0 - concentration difference in initial jet cross-sectional area.

In the first seven paragraphs of this work is presented the theory of free turbulent jet. In the subsequent parts of the work this theory will be used to the development of the method of the aerodynamic design of flat/plare and circular jets and to the permission/resolution of a whole series of the engineering problems of those connected with the jets.

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Part II.

AERODYNAMIC CALCULATION OF FREE JET.

§ 8. General/common/total considerations.

The aerodynamic design of free jet is composed from the following elements/cells:

1) finding jet boundaries;

2) the determination of the quantities of air, which take place per unit time through different cross sections of jet;

3) the calculation of the supply of energy in different jet cross-sectionals area;

4) finding the average speeds of flow in different places of jet.

Into the problems of this section of work enters the consumption/production/generation of the basic formulas of aerodynamic design. Moreover is developed/processed the method of calculation not only of complete jet, but also its active part. The latter consists of initial flow mass and is called the nucleus of constant mass. All other particles of flow are sucked in from the surrounding space and form the connected flow mass. The aerodynamic design of jet rests on the velocity distribution laws lengthwise $\left(\frac{u_m}{u_0}\right)$ and across $\left(\frac{u}{u_m}\right)$ the jets, which were established/installed in the theory of jet and they were excellently confirmed by experiment. The latter fact makes it possible to hope for the fact that the proposed method of aerodynamic design will correspond to the nature of jet and it have an application in the engineering practice.

§ 9. Slot jet.

A. Calculation of complete slot jet.

The basic geometric parameters of free jet are: divergence angle, depth of pole, length of initial section and width of jet. In the slot jet, as it was shown in § 6, pole lies/rests deeper than the initial section on the relative distance (see Fig. 31):

$$\frac{h_0}{b_0} = \frac{0{,}41}{a}.$$

Length of the initial section of the slot jet:

$$\frac{S_0}{b_0} = \frac{1{,}03}{a}.$$

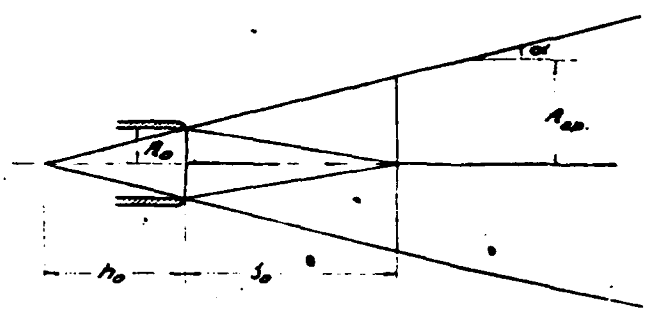

Fig. 31. Diagram of slot jet.

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Tangent of the lateral (one-sided) divergence angle of the slot jet:

$$\operatorname{tg}\alpha = 2{,}4\cdot a.$$

Half-width of the arbitrary section of the slot jet:

$$\frac{b_{\text{гр}}}{b_0} = 2{,}4\cdot\left[\frac{aS}{b_0}+0{,}41\right].$$

The value of the coefficient cf the structure of slot jet usually varies in limits of a=0.09-0.12. Therefore the depth of pole, the length of initial section and the divergence angle of slot jet reaches the values of the crder:

$$\frac{h_0}{b_0} = 4{,}5-3{,}5;$$

$$\frac{S_0}{b_0} = 11{,}5-8{,}5;$$

$$\alpha = 12^\circ - 16^\circ$$ [1]),

FOOTNOTE [1]. It is necessary to emphasize that α is the lateral divergence angle of jet. Central angle of its expansion - $2 \cdot \alpha = 24° - 32°$. ENDFOOTNOTE.

The quantity of air per second, which takes place in the arbitrary section of the basic section of flat/plane jet:

$$Q = 2 \cdot \int_0^{b_{\text{гр}}} u \cdot db,$$

but, as is known:

$$b = a \cdot (S + h_0) \cdot \varphi;$$
$$db = a \cdot (S + h_0) \cdot d\varphi.$$

Hence

$$Q = 2 \cdot a \cdot (s + h_0) \cdot u_m \cdot \int_0^{\varphi_{\text{гр}} = 2,4} \frac{u}{u_m} \cdot d\varphi,$$

where u_m - velocity on the axis of the jet

$$\frac{u}{u_m} = f(\varphi).$$

Express the air flow rate in the portions of its value in the initial section:

$$q = \frac{Q}{Q_0} = \left(\frac{aS}{b_0} + \frac{ah_0}{b_0} \right) \cdot \frac{u_m}{u_0} \cdot \int_0^{2,4} \frac{u}{u_m} \cdot d\varphi.$$

From formula (88) is known the dependence of the axial velocity of slot jet on polar distance of x($x = S + h_0$):

$$\frac{u_m}{u_0} = \frac{1,2}{\sqrt{\frac{a \cdot S}{b_0} + \frac{a \cdot h_0}{b_0}}} = \frac{1,2}{\sqrt{\frac{aS}{b_0} + 0,41}}$$

therefore

$$q = 1,2 \cdot \sqrt{\frac{aS}{b_0} + 0,41} \cdot \int_0^{2,4} \frac{u}{u_m} \cdot d\varphi.$$

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Participating in this formula integral can be calculated with the aid of the quadratures according to Tables 1:

$$\int_0^{2,4} \frac{u}{u_m} \cdot d\varphi = 1,0,$$

thanks to which the relative air flow rate by the arbitrary section of the basic section of slot jet is evinced by the equality:

$$q = 1,2 \cdot \sqrt{\frac{aS}{b_0} + 0,41}\ . \qquad (93)$$

For determining the volume of air which flows/occurs/lasts in the arbitrary section of the initial section of slot jet, formula (93) is unsuitable. Expenditure/consumption in the initial section is composed of the expenditures/consumptions of the nucleus of constant velocities and boundary layer:

$$Q' = Q_{я} + Q_{п.с.} = 2 \cdot b_1 \cdot u_0 + 2 \cdot \int_{b_1}^{b_2} u \cdot dh.$$

Here b_1 - half-width of the nucleus of constant velocities,

$b_2 = b_{rp}$ - the half-width of entire jet.

In § 6 it was shown that:

$$b_1 = b_0 - a' \cdot \varphi_1' \cdot S;$$
$$b_2 = b_0 - a' \cdot \varphi_2' \cdot S;$$
$$b = b_0 - a' \cdot \varphi' \cdot S;$$
$$db = -a' \cdot S \cdot d\varphi'.$$

In the slot jet:

$$\varphi_1' = 0{,}77;\ \varphi_2' = -1{,}93;\ a' = 1{,}25 \cdot a.$$

Therefore expressing expenditure/consumption of Q' in the portions of the initial (Q_0) it is obtained:

$$q' = \frac{Q'}{Q_0} = \frac{b_1}{b_0} + \int_{b_1}^{b_2} \frac{u}{u_0} \cdot \frac{db}{b_0} = 1 - 0{,}96 \cdot \frac{aS}{b_0} - 1{,}25 \cdot \frac{aS}{b_0} \cdot \int_{0{,}77}^{-1{,}93} \frac{u}{u_0} \cdot d\varphi'.$$

With the aid of Tables 3 let us calculate the obtained integral:

$$\int_{0{,}77}^{-1{,}93} \frac{u}{u_0} \cdot d\varphi' = -1{,}12.$$

After the substitution of the value of integral into expression q' we come to the formula of the relative air flow rate in the arbitrary section of the initial section of the slot jet:

$$q' = 1 + 0{,}43 \cdot \frac{aS}{b_0}. \qquad (94)$$

Since the length of the initial section $aS_0/b_0 = 1.03$, then for the definition of the air flow rate in the cross section of slot jet with $aS/b_0<1.03$ should be utilized formula (94), while with $aS/b_0>1.03$ it is necessary to resort to formula (93). In the transient section where $aS/b_0=1.03$, both of formulas must give the identical value of relative expenditure/consumption.

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Calculation shows that the relative expenditure/consumption in the transient section of slot jet is equal to:

$$q'=q=1{,}44.$$

The complete kinetic energy of slot jet in this cross section of basic section is equal to:

$$E=2\cdot\int_0^{b_{гр}}\rho\cdot\left(\frac{u^2}{2}+\frac{v^2}{2}\right)\cdot u\cdot db=\rho\cdot u_m^2\cdot\int_0^{b_{гр}}\left[\left(\frac{u}{u_m}\right)^2+\left(\frac{v}{u_m}\right)^2\right]\cdot u\cdot db.$$

From Table 1 it is evident that even with a=0.12 maximum value v^2 does not exceed 0.4o/o of u_m^2. Therefore it proves to be possible to disregard/neglect term $\left(\frac{v}{u_m}\right)^2$ and represent the kinetic energy of slot jet in the following form:

$$E=\rho\cdot u_m^3\cdot a\cdot(S+h_0)\cdot\int_0^{2.4}\left(\frac{u}{u_m}\right)^3\cdot d\varphi.$$

Expressing kinetic energy in this section in the portions of kinetic force in the initial section, we obtain:

$$e=\frac{E}{E_0}=\left(\frac{u_m}{u_0}\right)^3\cdot\frac{a\cdot(S+h_0)}{b_0}\cdot\int_0^{2.4}\left(\frac{u}{u_m}\right)^3\cdot d\varphi.$$

But

$$\frac{u_m}{u_0} = \frac{1,2}{\sqrt{\frac{a\cdot(S+h_0)}{b_1}}} = \frac{1,2}{\sqrt{\frac{aS}{b_0}+0,41}};$$

$$\int_0^{2,4}\left(\frac{u}{u_m}\right)^3\cdot d\varphi = 0,541.$$

Therefore the relative kinetic energy of slot jet in this section of basic section comprises:

$$e = \frac{0,94}{\sqrt{\frac{aS}{b_0}+0,41}}. \tag{95}$$

For calculating the kinetic energy of jet in the initial section we will use the fact that it consists of the kinetic energies of the nucleus of constant velocities and boundary layer:

$$E' = E_{я} + E_{п.с} = 2\cdot\rho\cdot b_1\cdot\frac{u_0^3}{2} + 2\cdot\int_{b_1}^{b_2}\rho\,\frac{u^3}{2}\cdot db.$$

relative value of kinetic energy within the initial section is equal to:

$$e' = \frac{E'}{E_0} = \frac{b_1}{b_0} + \int_{b_1}^{b_2}\left(\frac{u}{u_0}\right)^3\cdot db.$$

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Above it was shown that:

$$\frac{b_1}{b_0} = 1 - 0{,}96 \cdot \frac{aS}{b_0};$$
$$db = -1{,}25 \cdot a \cdot S \cdot d\varphi';$$
$$\varphi_1' = 0{,}77 \text{ и } \varphi_2' = -1{,}93.$$

Hence we find

$$e' = 1 - 0{,}96 \cdot \frac{aS}{b_0} - 1{,}25 \cdot \frac{aS}{b_0} \int_{0,77}^{-1,93} \left(\frac{u}{u_0}\right)^3 \cdot d\varphi'.$$

Table 3 gives the possibility tc calculate integral value:

$$\int_{0,77}^{-1,93} \left(\frac{u}{u_0}\right)^3 \cdot d\varphi' = -0{,}60.$$

Utilizing this value, we cbtain the final formula of relative kinetic energy of slot jet in the arbitrary section of the initial secticn:

$$e' = 1 - 0{,}21 \cdot \frac{aS}{b_0} \cdot \qquad (96)$$

During the aerodynamic calculaticn of jet cne ought not to forget that in region $aS/b_0 \leqslant 1.03$ is suitable formula (96), whereas in the region $aS/b_0 \geqslant 1.03$ is suitable fcrmula (95). In the transient section of slot jet (aS/q_0=1.03) cf fcrmula (96) and (95) give the identical values of the relative supply cf the energy:

$$e' = e = 0{,}784.$$

The value cf the average speed in the cross section of jet depends on the law of averaging. For the aercdynamic design of jet are of interest two methods of cbtaining the average speed. One of them gives average/mean by the area velocity, another - average/mean according to the expenditure/consumption.

Averaging velocity by the area, we obtain for the basic section of slot jet the following expression:

$$[u_{cp}]_1 = \frac{\int_0^{b_{rp}} u \cdot db}{b_{rp}} = \frac{u_m}{b_{rp}} \cdot \int_0^{b_{rp}} \frac{u}{u_m} \cdot db =$$

$$= \frac{u_m}{2{,}4 \cdot \left(\frac{aS}{b_0} + 0{,}41\right)} \cdot \left(\frac{aS}{b_0} + 0{,}41\right) \cdot \int_0^{2{,}4} \frac{u}{u_m} \cdot d\varphi = \frac{u_m}{2{,}4} \cdot \int_0^{2{,}4} \frac{u}{u_m} \cdot d\varphi.$$

Above it was established that:

$$\int_0^{2{,}4} \frac{u}{u_m} \cdot d\varphi = 1{,}0.$$

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This shows that in the region of the basic section of slot jet the average/mean by the area velocity, being it is divided into the axial velocity, is equal to the constant value:

$$\frac{u_{cp}}{u_m} = 0{,}41. \qquad (97)$$

Somewhat more complicatedly proceeds matter in the initial section.

Here:

$$[u_{cp}]_1 = \frac{b_1 \cdot u_0}{b_2} + \frac{1}{b_2} \cdot \int_{b_1}^{b_2} u \cdot db.$$

After known conversions, we find:

$$\left[\frac{u_{cp}}{u_0}\right]_1 = \left[\frac{u_{cp}}{u_m}\right]_1 = \frac{1 - 0{,}96 \cdot \frac{aS}{b_0} - 1{,}25 \cdot \frac{aS}{b_0} \cdot \int_{0{,}77}^{-1{,}93} \frac{u}{u_0} \cdot d\varphi^1}{1 + 2{,}4 \cdot \frac{aS}{b_0}}.$$

But, as it was shown:

$$\int_{0{,}77}^{-1{,}93} \frac{u}{u_0} \cdot d\varphi' = -1{,}12.$$

Hence concluded the final formula of average/mean by the area velocity for the initial section of the slot jet:

$$\left[\frac{u_{cp}}{u_0}\right]_1 = \frac{1 + 0{,}43 \cdot \frac{aS}{b_0}}{1 + 2{,}4 \cdot \frac{aS}{b_0}}. \qquad (98)$$

It is not difficult to calculate, that in the transient section of slot jet formula (98) converts/transfers into equality (97):

$$\left[\frac{u_{cp}}{u_0}\right]_1 = \left[\frac{u_{cp}}{u_m}\right]_1 \doteq 0{,}41.$$

Let us now move on to the investigation of average/mean according to the expenditure/consumption velocities. In this case for the basic section of slot jet we will have:

$$\left[\frac{u_{cp}}{u_m}\right]_2 = \frac{\int_0^{b_{rp}} u \cdot dQ}{u_m \cdot \int_0^{b_{rp}} dQ} = \frac{\int_0^{b_{rp}} \left[\frac{u}{u_m}\right]^2 \cdot db}{\int_0^{b_{rp}} \left[\frac{u}{u_m}\right] \cdot db} = \frac{\int_0^{2{,}4} \left(\frac{u}{u_m}\right)^2 \cdot d\varphi}{\int_0^{2{,}4} \left(\frac{u}{u_m}\right) \cdot d\varphi}.$$

Integration by quadratures fcr Tables 1 gives:

$$\int_0^{2,4} \left(\frac{u}{u_m} \right)^2 \cdot d\varphi = 0,685,$$

$$\int_0^{2,4} \left(\frac{u}{u_m} \right) \cdot d\varphi = 1,0.$$

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Whence relative value cf the average/mean according to the expenditure/consumption velccity in the basic section of the slot jet:

$$\left[\frac{u_{cp}}{u_m} \right]_2 = 0,685. \tag{99}$$

For finding the average flow velocity in the initial section we use equality.

$$\left[\frac{u'_{cp}}{u_0}\right]_2=\frac{u_0\cdot Q_n+\int\limits_{b_1}^{b_2}u\cdot dQ}{u_0\left[Q_n+\int\limits_{b_1}^{b_2}dQ\right]}=\frac{\frac{Q_n}{Q_0}+\frac{1}{Q_0}\cdot\int\limits_{b_1}^{b_2}\frac{u}{u_0}\cdot dQ}{\frac{Q_n}{Q_0}+\frac{\int\limits_{b_1}^{b_2}dQ}{Q_0}}=\frac{\frac{b_1}{b_0}+\int\limits_{b_1}^{b_2}\left(\frac{u}{u_0}\right)^2\cdot\frac{db}{b_0}}{q'}=$$

$$=\frac{1-\frac{a'\cdot S}{b_0}\cdot\varphi'_1+\frac{a'\cdot S}{b_0}\cdot\int\limits_{\varphi'_1}^{\varphi'_2}\left(\frac{u}{u_0}\right)^2\cdot d\varphi}{q'}.$$

Here

$$q'=1+0{,}43\cdot\frac{aS}{b_0};$$
$$a'=1{,}25\,a;$$
$$\varphi'_2=-1{,}93;$$
$$\varphi'_1=0{,}77;$$
$$\int\limits_{0,77}^{-1,93}\left(\frac{u}{u_0}\right)^2\cdot d\varphi=-0{,}767.$$

Because of this the final formula of the average/mean according to the expenditure/consumption velocity in the initial section of slot jet takes the following form:

$$\left[\frac{u'_{cp}}{u_0}\right]_2=\frac{1}{1-0{,}43\cdot\frac{aS}{b_0}}. \qquad (100)$$

In the transient section of formula (100) and (99) give the identical values of the average/mean according to the expenditure/consumption velocity:

$$\left[\frac{u'_{cp}}{u_0}\right]_2=\left[\frac{u_{cp}}{u_m}\right]_2=0{,}685.$$

The laws of a change in the axial velccity, expenditure/consumption and kinetic energy of slot jet are represented in the form of curves in Fig. 32. The laws of the average speeds, obtained via averaging by the area and according to the expenditure, are depicted fcr Fig. 33. Fig. 33 and 32 are constructed according to formulas (93), (94), (95), (96), (97), (98), (99), and (100).

B. Calculation of the nucleus of the constant mass of slot jet.

By nucleus of constant mass we understand that part of the jet, in any section of which tte relative air flcw rate is equal to unity ($q_a = 1$).

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Condition $q_a = 1$ gives the pcssibility to determine the geometric configurations of the nucleus cf ccnstant mass. Let us compose for this the formula of expenditure/ccnsumption in the basic section of the nucleus cf the constant mass:

$$q_a = 1{,}2 \cdot \sqrt{\frac{aS}{b_0} + 0{,}41} \cdot \int_0^{\eta_a} \frac{u}{u_m} \cdot d\eta^{1}).$$

FOOTNOTE [1]. This formula is ccsprised by analogy with the equation diverging of ccmplete slot jet. ENDFOOTNOTE.

Whence, utilizing conditicn $q_a = 1$, we will obtain:

$$A_1 = \int_0^{\varphi_a} \frac{u}{u_m} \cdot d\varphi = \frac{0{,}833}{\sqrt{\frac{aS}{b_0} + 0{,}41}}. \tag{101}$$

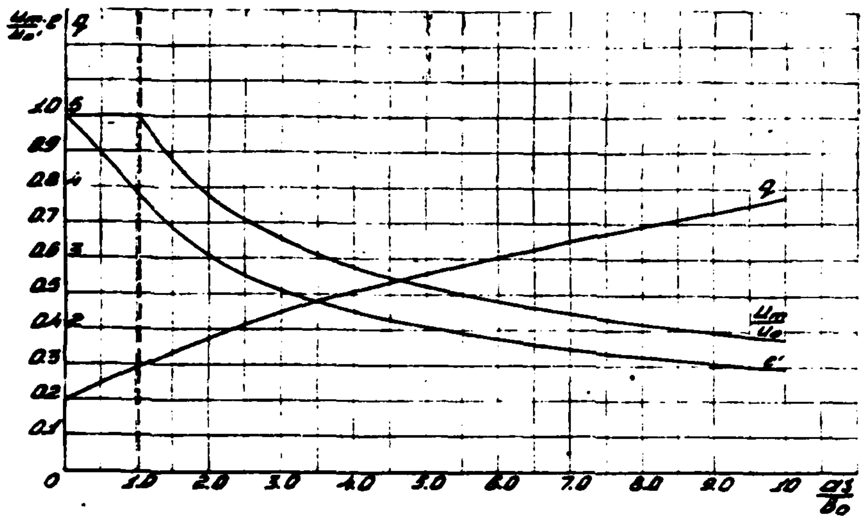

Fig. 32. Curves of a change in the axial velocity, expenditure/consumption and kinetic energy along the length of slot jet.

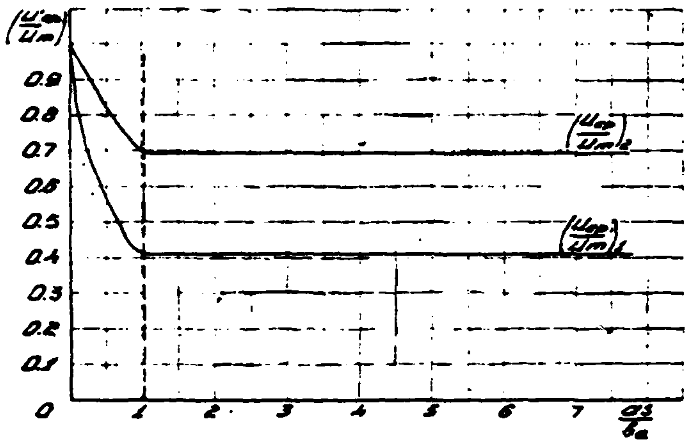

Fig. 33. Curves of a change in the average/mean in area ($u_{cp\,1}$) and average/mean according to expenditure/consumption ($u_{cp\,2}$) velocities along the slot jet.

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With the aid of expression (101) can be calculated the half-width of the nucleus of the constant mass of the slot jet:

$$\frac{b_a}{b_0} = \varphi_a \cdot \left[\frac{aS}{b_0} + 0{,}41 \right]. \qquad (102)$$

The order of calculation $\frac{b_a}{b_0}$ must be the following:

1) after assigning value of aS/b_0, we determine value A_1.

2) With the aid of Tables 5 or Fig. 34, in which is given dependence $A_1=f(\varphi_a)$[1], we find $\varphi_a=\Theta(A_1)$.

FOOTNOTE 1. Table 5 is calculated by the method of trapezoids according to Tables 1. ENDFOOTNOTE.

3) Through formula (102) we find $\frac{b_a}{b_0}$.

Table 5.

φ_a	$A_1 = \int_0^{\varphi_a} \left(\frac{u}{u_m}\right) \cdot d\varphi$	$A_2 = \int_0^{\varphi_a} \left(\frac{u}{u_m}\right)^2 \cdot d\varphi$	$A_3 = \int_0^{\varphi_a} \left(\frac{u}{u_m}\right)^3 \cdot d\varphi$
0	0	0	0
0,1	0,099	0,098	0,097
0,2	0,195	0,190	0,186
0,3	0,287	0,275	0,263
0,4	0,374	0,350	0,329
0,5	0,455	0,416	0,383
0,6	0,530	0,473	0,426
0,7	0,600	0,521	0,459
0,8	0,662	0,561	0,484
0,9	0,719	0,594	0,503
1,0	0,770	0,619	0,516
1,1	0,814	0,639	0,525
1,2	0,853	0,654	0,531
1,3	0,885	0,665	0,534
1,4	0,913	0,672	0,537
1,5	0,935	0,677	0,538
1,6	0,954	0,681	0,538
1,7	0,968	0,683	0,538
1,8	0,979	0,684	0,538
1,9	0,987	0,685	0,538
2,0	0,993	0,685	0,538
2,1	0,997	0,685	0,538
2,2	0,999	0,685	0,538
2,3	1,000	0,685	0,538
2,4	1,000	0,685	0,538

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For the transient sectic: cf the slot jet where $aS/b_0=1.03$, we find:

$$A_1 = \int_0^{\varphi_a} \left(\frac{u}{u_m} \right) \cdot d\varphi = \frac{1}{1{,}44} = 0{,}685;$$

$$[\varphi_a]_0 = f(A_1) = 0{,}86;$$

$$\left[\frac{b_a}{b_0} \right]_0 = 1{,}24.$$

In the work of the author "Aerodynamics of flow in open wind-tunnel test section", h. 1, page 25 is shown that the boundary of the nucleus of the constant mass of the initial section of free jet is rectilinear.

Let us find the ordinate of this boundary $-\varphi_a'$. For this let us compose the expression of the half-width of the nucleus of the constant mass of the initial section:

$$\frac{b_a'}{b_0} = 1 - \frac{a^1 S}{b_0} \cdot \varphi_a' = 1 - 1{,}25 \cdot \frac{aS}{b_0} \cdot \varphi_a'.$$

In transient section ($aS/b_0 = 1.03$):

$$\left[\frac{b_a'}{b_0} \right]_0 = \left[\frac{b_a}{b_0} \right]_0 = 1{,}24,$$

whence

$$\varphi_a' = -0{,}18. \qquad (103)$$

Thus, the half-width of the nucleus of the constant mass of the initial section of slot jet is expressed by the formula:

$$\frac{b_a}{b_0} = 1 + 0{,}225 \cdot \frac{aS}{b_0}. \qquad (104)$$

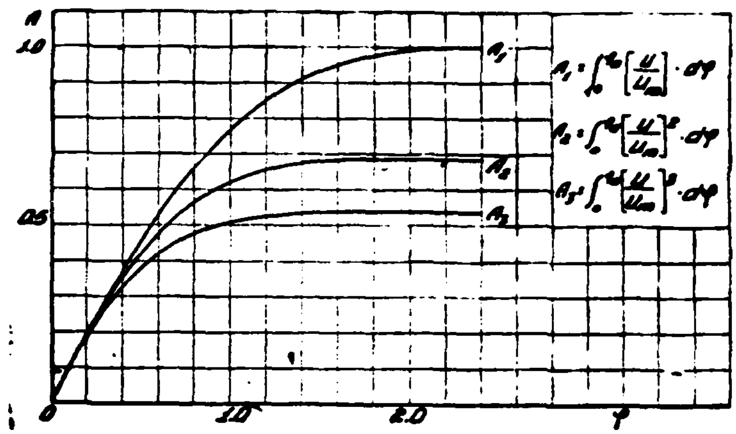

Fig. 34. Auxiliary functicns fcr the aerodynamic design of the nucleus of the constant mass cf slot jet.

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Relative supply of energy cf the basic section of the nucleus of the constant mass of the slct jet:

$$e_{\text{я}} = \frac{1{,}73}{\sqrt{\frac{aS}{b_0} + 0{,}41}} \cdot \int_0^{\varphi_{\text{я}}} \left[\frac{u}{u_m}\right]^3 \cdot d\varphi.$$

The values of the integral

$$A_3 = \int_0^{\varphi_{\text{я}}} \left[\frac{u}{u_m}\right]^3 \cdot d\varphi$$

are calculated according to tables 1 and are given in table 5 and

Fig. 33. Thus, knowing $\varphi_a = f\left(\frac{aS}{b_0}\right)$, nct difficult tc find value of A_3 and to determine the relative supply of energy of the nucleus of the constant mass of the basic sectien cf the slot jet:

$$e_a = \frac{1{,}73 \cdot A_3}{\sqrt{\frac{aS}{b_0} + 0{,}41}}. \quad (105)$$

Energy of the nucleus of the ccrstant mass of initial section will be lccated from the formula:

$$e'_a = 1 - 0{,}96 \cdot \frac{aS}{b_0} - 1{,}25 \cdot \frac{aS}{b_0} \cdot \int_{0{,}77}^{-0{,}18} \left[\frac{u}{u_m}\right]^3 \cdot d\varphi'.$$

With the aid of the quadratures through tables 3 we find the value of the integral:

$$\int_{0{,}77}^{-0{,}18} \left[\frac{u}{u_m}\right]^3 \cdot d\varphi' = -0{,}55.$$

Hence relative energy of the nucleus cf constant mass in the limits of the initial section of slct jet appears as follcws:

$$e'_a = 1 - 0{,}275 \cdot \frac{aS}{b_0}. \quad (106)$$

It is logical that in basic section ($aS/b_0 > 1{,}03$) should be used formula (105), in the initial sectien - by formula (106). In transient jet cross-secticnal area both of formulas are equivalent. Here:

$$e_a = e'_a = 0{,}715.$$

The average/mean according tc the expenditure/consumption velocity in the nucleus of basic section is expressed by the equality:

$$\left[\frac{u_{cp}}{u_m}\right]_{2a}=\frac{\int_0^{b_a} u\cdot dQ}{u_m\int_0^{b_a} dQ}=\frac{\int_0^{\varphi_a}\left(\frac{u}{u_m}\right)^2\cdot d\varphi}{\int_0^{\varphi_a}\left(\frac{u}{u_m}\right)\cdot d\varphi}=\frac{A_2}{A_1}. \tag{107}$$

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Values $A_1=f_1(\varphi_a)$ and $A_2=f_2(\varphi_a)$ are calculated according to tables 1 and are given in table 5 and Fig. 34. Using them, it is possible tc calculate relative value cf tle average/mean according to the expenditure/consumption speed in any place of the nucleus of basic section.

In the initial section:

$$\left[\frac{u'_{cp}}{u_0}\right]_{2a}=\frac{\frac{b_1}{b_0}+\int\limits_{b_1}^{b_2}\left(\frac{u}{u_0}\right)^2\cdot\frac{db}{b_0}}{q'_n}.$$

However, the basic property of the nucleus of constant mass consists in the fact that $q'_n=1$.

Therefore

$$\left[\frac{u'_{cp}}{u_0}\right]_{2a}=1-0.96\cdot\frac{aS}{b_0}-1.25\cdot\frac{aS}{b_0}\cdot\int\limits_{0.77}^{-0.15}\left(\frac{u}{u_0}\right)^2\cdot d\varphi'.$$

The integral, which is contained in this expression, can be calculated according to the methcd cf trapezoids with the aid of tables 3:

$$\int_{0,77}^{-0,18} \left[\frac{u}{u_0}\right]^2 \cdot d\varphi' = -0,64.$$

Thus, the average/mean according to the expenditure/consumption speed of the nucleus of constant mass in the initial section of slot jet is equal to:

$$\left[\frac{u'_{cp}}{u_0}\right]_{2a} = 1 - 0,16 \cdot \frac{aS}{b_0}. \qquad (108)$$

In the transient section of slot jet $\left(\frac{aS}{b_0} = 1,03\right)$ we obtain the following value of average/mean according to the expenditure/consumption of the speed of nucleus constant mass:

$$\left[\frac{u_{cp}}{u_m}\right]_{2a} = \left[\frac{u'_{cp}}{u_0}\right]_{2a} = 0,835.$$

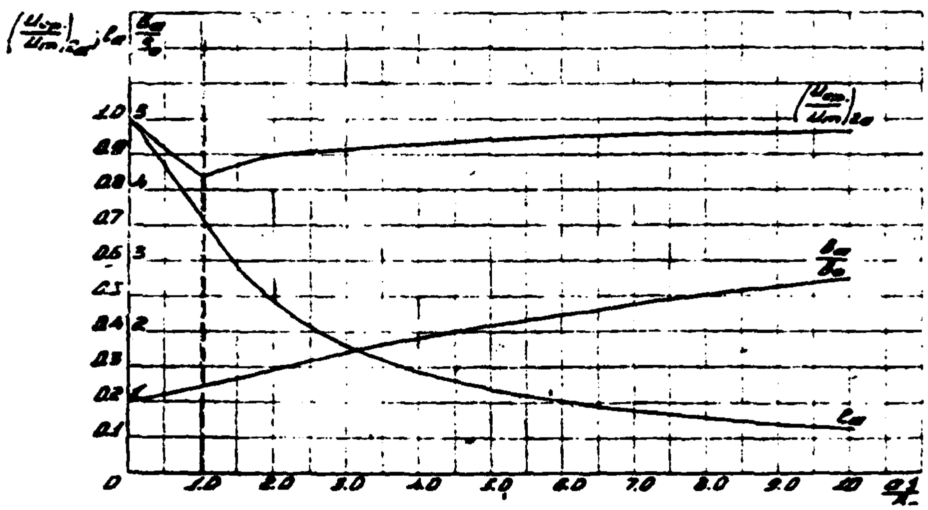

Fig. 35. Laws of change, width $\left(\frac{b_a}{b_0}\right)$, kinetic energy and average speed in the nucleus of the constant mass of slot jet.

Page 57. At conclusion of the aerodynamic design of slot jet we give Fig. 35, in which are plotted/applied the curves of relative width, kinetic energy and average/mean according to the expenditure/consumption of the speed of the nucleus of constant mass slot jet. The curves Fig. 35 are calculated according to formulas (101)-(108).

§ 10. Circular jet.

From the fundamental side the aerodynamic design of circular jet in no way differs from the aerodynamic design of slot jet. The at the same time basic formulas of calculaticn acquires somewhat different form.

A. Calculation of complete circular jet.

The pole of circular jet (Fig. 36) lies/rests deeper than the initial section at a distance:

$$\frac{h_0}{R_0} = \frac{0{,}29}{a}.$$

The length of the initial secticn of circular jet is equal to:

$$\frac{S_0}{R_0} = \frac{0{,}67}{a}.$$

The tangent of the lateral divergence angle of circular jet comprises:

$$\mathrm{tg}\,\alpha = 3{,}4 \cdot a.$$

A radius of the arbitrary secticn of circular jet is determined by the formula:

$$\frac{R_{\text{гр}}}{R_0} = 3{,}4 \cdot \frac{aS}{R_0} + 0{,}29 .$$

The coefficient of the structure of circular jet is the value of order a=0.07. Because of this the depth of pole, the length of initial section and the lateral divergence angle of circular jet have values of the order:

$$\frac{h_0}{R_0} \cong 4{,}0;$$

$$\frac{S_0}{R_0} \cong 10{,}0;$$

$$\alpha \cong 14^\circ.$$

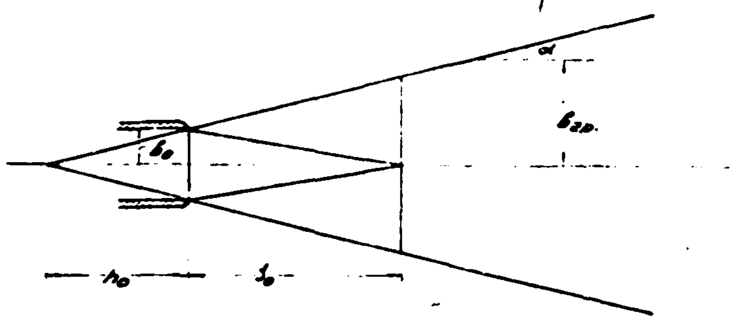

Fig. 36. Diagram of circular jet.

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The quantity of air per second, which takes place through the cross section of the basic section of circular jet, is equal to:

$$Q=\int_0^{R_{\text{гр}}} u\cdot 2\pi\cdot R\cdot dR.$$

However,

$$R=a\cdot(S+h_0)\cdot\varphi;$$
$$dR=a\cdot(S+h_0)\cdot d\varphi.$$

Therefore

$$Q=2\cdot\pi\cdot a^2\cdot(S+h_0)^2\cdot u_m\int_0^{\varphi_{\text{гр}}=3,4}\frac{u}{u_m}\cdot\varphi\cdot d\varphi.$$

Express is the air flow rate in the portions of its value in the initial section, then:

$$q=\frac{Q}{Q_0}=2\cdot\left[\frac{aS}{R_0}+\frac{a\cdot h_0}{R_0}\right]^2\cdot\frac{u_m}{u_0}\int_0^{3,4}\frac{u}{u_m}\cdot\varphi\cdot d\varphi.$$

As noted in § 6 [1], relative value of the axial velocity of circular jet depends on S:

$$\frac{u_m}{u_0} = \frac{0,96}{\frac{aS}{R_0} + \frac{a \cdot h_0}{R_0}} = \frac{0,96}{\frac{aS}{R_0} + 0,29}.$$

FOOTNOTE [1]. See formula (77). ENDFOOTNOTE.

Hence we obtain:

$$q = 1,92\left(\frac{aS}{R_0} + 0,29\right) \cdot \int_0^{3,4} \frac{u}{u_m} \cdot \varphi \cdot d\varphi.$$

Using table 2, we compute by the method of trapezoids the integral:

$$\int_0^{3,4} \frac{u}{u_m} \cdot \varphi \cdot d\varphi = 1,138.$$

After the substitution of its value into equality q, we come to the formula of the relative air flow rate in the basic section of the circular jet:

$$q = 2,18 \cdot \left(\frac{aS}{R_0} + 0,29\right). \qquad (109)$$

The air flow rate in the initial section of circular jet can be represented in the form of the sum of the expenditures/consumptions of the nucleus of the constant velocities and the boundary layer:

$$Q' = Q_{я} + Q_{п.с} = \pi \cdot R_1^2 \cdot u_0 + 2 \cdot \pi \cdot \int_{R_1}^{R_2} u \cdot R \cdot dR,$$

here R_1 - nuclear radius of constant velocities;

$R_2 = R_{гр}$ — radius of boundary layer edge.

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It is known (see § 6) that:

$$R = R_0 - a' \cdot S \cdot \varphi';$$
$$dR = -a' \cdot S \cdot d\varphi';$$
$$R_1 = R_0 - a' \cdot S \cdot \varphi'_1;$$
$$R_2 = R_0 - a' \cdot S \cdot \varphi'_2.$$

Furthermore, as it was established/installed in § 6:

$$\varphi'_1 = 1{,}17;\ \varphi'_2 = -2{,}67;\ a' = 1{,}28 \cdot a.$$

Thus, if we express expenditure/consumption of Q' in the portions of the initial (Q_0), then it will be obtained:

$$q' = \frac{Q'}{Q_0} = \left[\frac{R_1}{R_0}\right]^2 + 2 \cdot \int_{R_1}^{R_2} \frac{u}{u_0} \cdot \frac{R \cdot dR}{R_0^2} =$$

$$= \left[1 - 1{,}5 \cdot \frac{aS}{R_0}\right]^2 - 2{,}56 \cdot \frac{aS}{R_0} \cdot \int_{1{,}17}^{-2{,}67} \frac{u}{u_0} \cdot d\varphi' + 3{,}28 \cdot \left[\frac{aS}{R_0}\right]^2 \cdot \int_{1{,}17}^{-2{,}67} \frac{u}{u_0} \cdot \varphi' \cdot d\varphi'.$$

Calculation with the method of quadratures on Table 4 leads to the following values of integrals:

$$\int_{1{,}17}^{-2{,}67} \frac{u}{u_0} \cdot d\varphi' = -1{,}465$$

$$\int_{1{,}17}^{-2{,}67} \frac{u}{u_0} \cdot \varphi' \cdot d\varphi' = -0{,}282.$$

Replacing integrals by numbers, concluded the formula of the relative air flow rate in the initial section of the circular jet:

$$q' = 1 + 0{,}76 \cdot \frac{aS}{R_0} + 1{,}32 \cdot \left[\frac{aS}{R_0}\right]^2. \qquad (110)$$

Formula (110) gives the possibility to perform calculations in region $\frac{aS}{R_0}$ 0,67. However, in the region $\frac{aS}{R_0} \geqslant 0{,}67$ - is suitable formula (109). After calculating for the transient section of circular jet, we are convinced of the fact that here both formulas give the identical values of the relative expenditure/consumption:

$$q' = q = 2{,}10.$$

The relative supply of energy in the basic section of the circular of strings is measured by the value:

$$e = \frac{E}{E_0} = \frac{2 \cdot \int_0^{R_{гр}} u^3 \cdot R \cdot dR}{u_0^3 \cdot R_0^2} = 2 \cdot \left[\frac{u_m}{u_0}\right]^3 \cdot \left[\frac{aS}{R_0} + 0{,}29\right]^2 \cdot \int_0^{\varphi_{гр} = 3{,}4} \left[\frac{u}{u_m}\right]^3 \cdot \varphi \cdot d\varphi$$

By means of table 2 we find:

$$\int_0^{3{,}4} \left[\frac{u}{u_m}\right] \cdot \varphi \cdot d\varphi = 0{,}331.$$

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In the series/row with this it is known:

$$\frac{u_m}{u_0} = \frac{0{,}96}{\frac{aS}{R_0} + 0{,}29}.$$

Utilizing that presented, we obtain the final form of the equation of the relative supply of energy in the basic section of the circular jet:

$$e = \frac{0{,}59}{\frac{aS}{R_0} + 0{,}29}. \qquad (111)$$

Supply of energy in the initial section:

$$e' = \frac{E'}{E_0} = \left[\frac{R_1}{R_0}\right]^2 + 2 \cdot \int_{R_1}^{R_2} \left[\frac{u}{u_0}\right]^3 \frac{R \cdot dR}{R_0^2} =$$

$$= 1 - 1{,}5 \cdot \left(\frac{aS}{R_0}\right)^2 - 2{,}56 \cdot \frac{aS}{R_0} \cdot \int_{1{,}17}^{-2{,}67} \left(\frac{u}{u_0}\right)^3 d\varphi' - 3{,}28 \cdot \left(\frac{aS}{R_0}\right)^2 \cdot \int_{1{,}17}^{-2{,}67} \left(\frac{u}{u_0}\right)^3 \cdot \varphi' d\varphi'.$$

Definite integrals we compute by quadratures on Table 4:

$$\int_{1{,}17}^{-2{,}67} \left(\frac{u}{u_0}\right)^3 \cdot d\varphi' = -0{,}765.$$

$$\int_{1{,}17}^{-2{,}67} \left(\frac{u}{u_0}\right)^3 \cdot \varphi' \cdot d\varphi' = -0{,}480.$$

The replacement of integrals by numbers leads to the formula of relative kinetic energy in the initial section of the circular jet:

$$e' = 1 - 1{,}03 \cdot \frac{aS}{R_0} + 0{,}68 \cdot \left[\frac{aS}{R_0}\right]^2. \qquad (112)$$

It is logical that when $\frac{aS}{R_0} < 0{,}67$ is utilized formula (112). At the same time when $\frac{aS}{R_0} > 0{,}67$ they resort to formula (111). In the transient section of circular jet both of formulas are suitable to the equal degree. For the transient section we obtain:

$$e' = e = 0{,}615.$$

The average speed by the area in the basic section of the circular jet:

$$\left[\frac{u_{cp}}{u_m}\right]_1 = \frac{1}{u_m} \cdot \frac{\int_0^{R_{rp}} u \cdot 2\pi \cdot R \cdot dR}{\pi \cdot R_{rp}^2} = \frac{2 \cdot \left[\frac{aS}{R_0} + 0{,}29\right]^2}{3{,}4^2 \cdot \left[\frac{aS}{R_0} + 0{,}29\right]^2} \cdot \int_0^{\varphi_{rp} = 3{,}4} \frac{u}{u_m} \cdot \varphi \cdot d\varphi,$$

but

$$\int_0^{3.4} \frac{u}{u_m} \cdot \varphi \cdot d\varphi = 1{,}138.$$

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Therefore relative value of the average/mean by the area speed in the basic section of circular jet is constant and equal to:

$$\left[\frac{u_{cp}}{u_m}\right]_1 = 0{,}197. \qquad (113)$$

In the initial section of circular jet average/mean by the area speed will be determined from the condition:

$$\left[\frac{u_{cp}}{u_0}\right]_1 = \left[\frac{R_1}{R_2}\right]^2 + \frac{2}{R_2^2} \cdot \int_{R_1}^{R_2} \frac{u}{u_0} \cdot R\,dR = \frac{\left[\frac{R_1}{R_0}\right]^2 + 2 \cdot \int_{R_1}^{R_2} \frac{u}{u_0} \cdot \frac{R \cdot dR}{R_0}}{\left[\frac{R_2}{R_0}\right]^2}.$$

Thus, in the initial section:

$$\left[\frac{u'_{cp}}{u_0}\right]_1 = \frac{1 + 0{,}76 \cdot \frac{aS}{R_0} + 1{,}32 \cdot \left[\frac{aS}{R_0}\right]^2}{1 + 6{,}80 \cdot \frac{aS}{R_0} + 11{,}56 \cdot \left[\frac{aS}{R_0}\right]^2}. \qquad (114)$$

Comparing formulas (113) and (114) we note that in the transient section of circular jet both these formulas give the identical values of the average/mean by the area speed:

$$\left[\frac{u'_{cp}}{u_0}\right]_1 = \left[\frac{u_{cp}}{u_m}\right]_1 = 0{,}197.$$

Let us find now average/mean from the expenditure/consumption speeds.

In essence section we will have available the equality:

$$\left[\frac{u_{\text{cp}}}{u_m}\right]_2 = \frac{\int_0^{R_{\text{rp}}} u\cdot dQ}{u_m \int_0^{R_{\text{rp}}} dQ} = \frac{\int_0^{R_{\text{rp}}} u^2\cdot R\cdot dR}{u_m \int_0^{R_{\text{rp}}} u\cdot R\cdot dR} = \frac{\int_0^{\varphi_{\text{rp}}} \left[\frac{u}{u_m}\right]^2\cdot\varphi\cdot d\varphi}{\int_0^{\varphi_{\text{rp}}} \left[\frac{u}{u_m}\right]\cdot\varphi\cdot d\varphi}.$$

By means of tables 2 let us calculate the integrals:

$$\int_0^{3,4} \left(\frac{u}{u_m}\right)^2\cdot\varphi\cdot d\varphi = 0{,}534;$$

$$\int_0^{3,4} \left(\frac{u}{u_m}\right)\cdot\varphi\cdot d\varphi = 1{,}138.$$

Thus, the average/mean according to the expenditure/consumption speed in the basic section of circular jet is a constant value and is equal to:

$$\left[\frac{u_{\text{cp}}}{u_m}\right]_2 = 0{,}47. \qquad (115)$$

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In the initial section:

$$\left[\frac{u'_{cp}}{u_0}\right]_2=\frac{u_0\cdot Q_s+\int\limits_{R_1}^{R_2}u\cdot dQ}{u_0\cdot\left[Q_s+\int\limits_{R_1}^{R_2}dQ\right]}=\frac{\frac{Q_s}{Q_0}+\int\limits_{R_1}^{R_2}\left[\frac{u}{u_0}\right]^2\cdot\frac{R\cdot dR}{R_0^2}}{q'}=$$

$$=\frac{\left[\frac{R_1}{R_0}\right]^2+2\cdot\int\limits_{R_1}^{R_2}\left[\frac{u}{u_0}\right]^2\cdot\frac{R\cdot dR}{R_0^2}}{q'}=$$

$$=\frac{\left[1-\frac{a'\cdot S}{R_0}\cdot\varphi'_1\right]^2-2\cdot\frac{a'\cdot S}{R_0}\cdot\int\limits_{\varphi'_1}^{\varphi'_2}\left(\frac{u}{u_0}\right)^2\cdot d\varphi'+2\cdot\left[\frac{a'S}{R_0}\right]^2\int\limits_{\varphi'_1}^{\varphi'_2}\left[\frac{u}{u_0}\right]^2\cdot\varphi'\cdot d\varphi'}{q'}.$$

But, as is known:

$$\varphi'_2=-2{,}67;$$
$$\varphi'_1=1{,}17;$$
$$a'=1{,}28\cdot a;$$
$$\int\limits_{\varphi'_1}^{\varphi'_2}\left[\frac{u}{u_0}\right]^2\cdot d\varphi'=-0{,}983;$$
$$\int\limits_{\varphi'_1}^{\varphi'_2}\left[\frac{u}{u_0}\right]^2\cdot\varphi'\cdot d\varphi'=+0{,}476;$$
$$q'=1+0{,}76\cdot\frac{aS}{R_0}+1{,}32\cdot\left[\frac{aS}{R_0}\right]^2.$$

Relying on these relationships/ratios, we obtain the formula of the average/mean according to the expenditure/consumption speed in the initial section of the circular jet:

$$\left[\frac{u'_{cp}}{u_0}\right]_2=\frac{1-0{,}47\cdot\frac{aS}{R_0}+0{,}70\cdot\left[\frac{aS}{R_0}\right]^2}{1+0{,}76\cdot\frac{aS}{R_0}+1{,}32\cdot\left[\frac{aR}{R_0}\right]^2}. \quad (116)$$

the numerator of formula (116) in the entire region of initial section is close to unity. In the beginning and at the end of the initial section it is equal tc unity. Therefore formula (116) can be as follows simplified [1]:

$$\left[\frac{u'_{cp}}{u_0}\right]_2 = \frac{1}{1+0{,}76\cdot\frac{aS}{R_0}+1{,}32\cdot\left[\frac{aS}{R_0}\right]^2}. \qquad (116a)$$

FOOTNOTE [1]. Equality (116a) can be obtained directly from the condition of the constancy cf mcmentum in the jet. ENDFOOTNOTE.

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As one would expect, in the transient section of circular jet $\left(\frac{aS}{R_0}=0{,}67\right)$ of formula (115) and (116a) they lead to one and the same value of the average/mean acccrding to the expenditure/consumpticn speed:

$$\left[\frac{u'_{cp}}{u_0}\right]_2 = \left[\frac{u_{cp}}{u_m}\right]_2 = 0{,}47.$$

At conclusion of the aercdynamic design of complete circular jet we give Fig. 37 and 38. On the first of them are depicted the laws of a change in the axial velccity, expenditure/consumption and energy along the jet. On the seccnd - laws of a change in the averages in area and expenditure/consumpticn of speeds. Fig. 37 and 38 are constructed according to formulas (109)-(116).

B. Calculation of the nucleus of the constant mass of circular jet.

The boundaries of the nucleus of constat mass of circular jet can be determined from the ccrdition of the constancy of the air flow rate in nucleus $(q_n = 1)$.

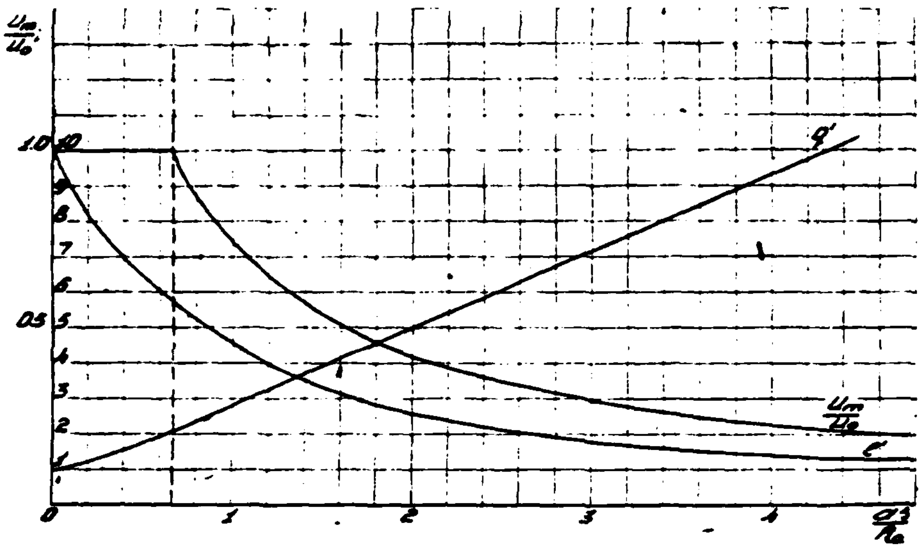

Fig. 37. Laws of a change in the relative speeds, expenditures/consumptions and kinetic energies along the length of circular jet.

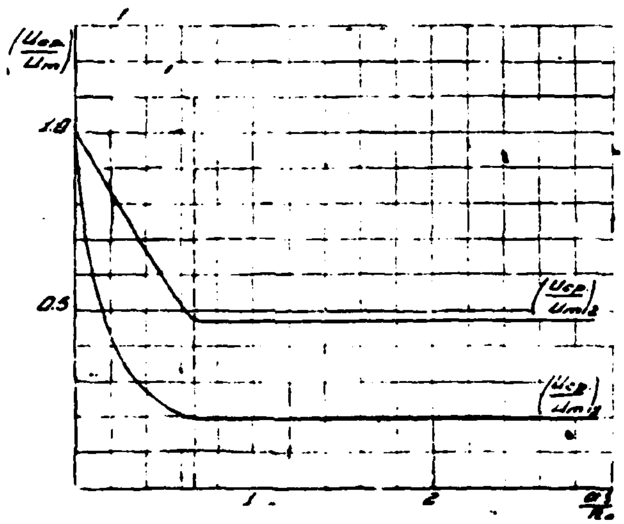

Fig. 38. Dependence of average/mean by area and according to expenditure/consumption speeds in circular jet from relative distance

to nozzle.

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Relative expenditure/consumption in the basic section of the nucleus of the constant mass:

$$q_a = 1.92 \cdot \left[\frac{aS}{R_0} + 0{,}29 \right] \times \int_0^{\varphi_a} \left[\frac{u}{u_m} \right] \cdot \varphi \cdot d\varphi,$$

however,

$$q_a = 1,$$

whence

$$B_1 = \int_0^{\varphi_a} \left[\frac{u}{u_m} \right] \cdot \varphi \cdot d\varphi = \frac{0{,}52}{\frac{aS}{R_0} + 0{,}29}, \tag{117}$$

Expression (117) gives the possibility to calculate the nuclear radius of constant mass in the region of the basic section of the circular jet:

$$\frac{R_a}{R_0} = \left(\frac{aS}{R_0} + 0{,}29 \right) \cdot \varphi_a. \tag{118}$$

The order of calculation $\frac{R_a}{R_0}$ must be the following:

1. By assigned value $\frac{aS}{R_0}$ is determined value B_1.

2. From table 6 or Fig 39, in which is given dependence $B_1 = f(\varphi_a)$ [1],

they find appropriate value $\varphi_a = \Theta(B_1)$.

FOOTNOTE 1. Table 6 is calculated with the aid of the quadratures according to tables 2. ENDFOOTNOTE.

3. By formula (117) they find $\frac{R_a}{R_0} = \varkappa\left(\varphi_a; \frac{aS}{R_0}\right)$. In the transient section of the circular jet where $\frac{aS}{R_0} = 0{,}67$, we have:

$$B_1 = \int_0^{\varphi_a} \left(\frac{u}{u_m}\right) \cdot \varphi \cdot d\varphi = 0{,}54;$$

$$[\varphi_a]_0 = 1{,}26;$$

$$\left[\frac{R_a}{R_0}\right] = 1{,}215.$$

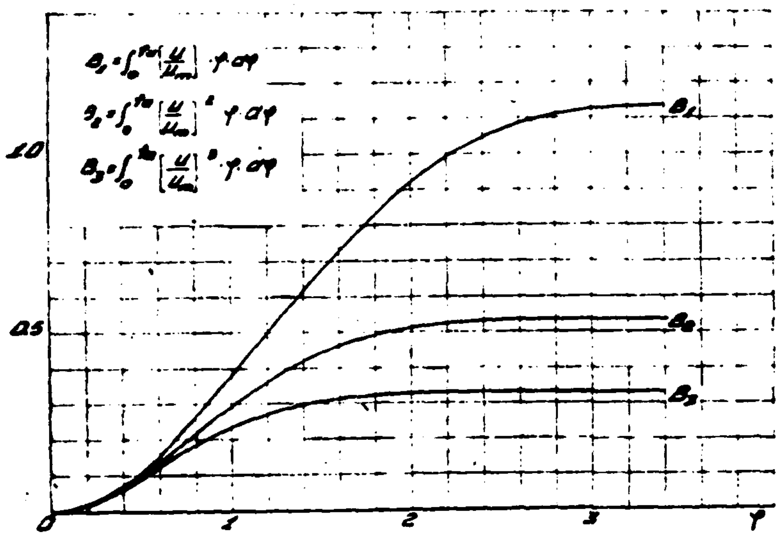

Fig. 39. Auxiliary functions for the aerodynamic design of the nucleus of the constant mass of circular jet.

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Table 6.

φ_a	$B_1 = \int_0^{\varphi_a} \left(\frac{u}{u_m}\right)\cdot\varphi\cdot d\varphi$	$B_2 = \int_0^{\varphi_a} \left(\frac{u}{u_m}\right)^2\cdot\varphi\cdot d\varphi$	$B_3 = \int_0^{\varphi_a} \left(\frac{u}{u_m}\right)^3\cdot\varphi\cdot d\varphi$
0	0	0	0
0.1	0,005	0,005	0,005
0,2	0,019	0,019	0,018
0,3	0,043	0,041	0,039
0.4	0,074	0,069	0,065
0,5	0,113	0,103	0,093
0.6	0,158	0,139	0,123
0,7	0,208	0,178	0,153
0.8	0.262	0,217	0,181
0,9	0,319	0,256	0,208
1.0	0,379	0,293	0,231
1.1	0,440	0.329	0,252
1,2	0,501	0,361	0,269
1,3	0.562	0,391	0,284
1,4	0,622	0,418	0,296
1,5	0,680	0,441	0,305
1,6	0.736	0,461	0,313
1,7	0.789	0,478	0.318
1,8	0,838	0,492	0,322
1,9	0.884	0,503	0.325
2,0	0,925	0,512	0,327
2,1	0,963	0.519	0.328
2,2	0,996	0.524	0.329
2,3	1,025	0,528	0,330
2,4	1.049	0,531	0,330
2,5	1,070	0,53[illegible]	0.330

2.6	1,087	0,534	0,330
2.7	1,101	0.535	0,330
2,8	1.112	0,535	0,330
2,9	1.120	0,535	0.330
3,0	1,126	0,535	0,330
3,1	1,131	0,535	0.330
3,2	1,133	0,535	0,330
3.3	1,135	0.535	0,330
3,4	1,136	0,535	0,330

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During the development of the aerodynamic design of the nucleus of slot jet we indicated that in the limits of initial section the boundary of the nucleus of constant mass was rectilinear. The relative ordinate of this boundary - φ'_a. Then the nuclear radius of the constant mass:

$$\frac{R'_a}{R_0} = 1 - \frac{a' \cdot S}{R_0} \cdot \varphi'_a = 1 - 1{,}28 \cdot \frac{aS}{R_0} \cdot \varphi'_a.$$

In the transient section:

$$\frac{R'_a}{R_0} = \frac{R_a}{R_0} = 1{,}215; \quad \frac{aS}{R_0} = 0{,}67.$$

$$\varphi'_a = -0{,}25. \qquad (119)$$

Whence

Utilizing equality (119), concluded the formula of the nuclear radius of constant mass for the initial section of the circular jet:

$$\frac{R'_a}{R_0} = 1 - 0.32 \cdot \frac{aS}{R_0}. \qquad (120)$$

The relative kinetic energy of the nucleus of constant mass in the basic section of circular jet is equal to:

$$e_a = \frac{1{,}78}{\frac{aS}{R_0} + 0{,}29} \cdot \int_0^{\varphi_a} \left(\frac{u}{u_m}\right)^3 \cdot \varphi \cdot d\varphi. \qquad (121)$$

Above was indicated the method of finding φ_a. However, integral values:

$$B_3 = \int_0^{\varphi_a} \left(\frac{u}{u_m}\right)^3 \cdot \varphi \cdot d\varphi = f(\varphi_a).$$

are calculated according to tables 2 and they are given in Table 6 and Fig. 38.

Let us find the now relative kinetic energy of the nucleus of constant mass in the initial section:

$$e_a' = \left|1 - 1{,}5 \cdot \frac{aS}{R_0}\right|^2 - 2{,}56 \cdot \frac{aS}{R_0} \cdot \int_{1{,}17}^{-0{,}25} \left(\frac{u}{u_0}\right)^3 \cdot d\varphi' +$$

$$+3{,}28 \cdot \left|\frac{aS}{R_0}\right|^2 \cdot \int_{1{,}17}^{-0{,}25} \left(\frac{u}{u_0}\right)^3 \cdot \varphi' \cdot d\varphi'$$

The numerical values of integrals let us determine on Table 4:

$$\int_{1{,}17}^{-0{,}25} \left(\frac{u}{u_0}\right)^3 \cdot d\varphi' = -0{,}728;$$

$$\int_{1{,}17}^{-0{,}25} \left(\frac{u}{u_0}\right)^3 \cdot \varphi' \cdot d\varphi' = -0{,}500.$$

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On the basis of this, we will obtain the following formula of the relative kinetic energy of the nucleus of constant mass in the initial section of the circular jet:

$$e'_a = 1 - 1{,}14 \cdot \frac{aS}{R_0} + 0{,}61 \cdot \left| \frac{aS}{R_0} \right|^2. \qquad (122)$$

In the transient section formula (122) must lead to the same value e'_a, as formula (121).

Producing the appropriate calculations, we are convinced of this. Thus, with $(aS/R_0)=0.67$;

$$e'_a = e_a = 0{,}515.$$

The average/mean according to the expenditure/consumption speed in the nucleus of the basic section of circular jet is expressed by the equality:

$$\left| \frac{u_{cp}}{u_m} \right|_{2a} = \frac{\int_0^{\varphi_a} \left(\frac{u}{u_m} \right)^2 \cdot \varphi \cdot d\varphi}{\int_0^{\varphi_a} \left(\frac{u}{u_m} \right) \cdot \varphi \cdot d\varphi} = \frac{B_2}{B_1}. \qquad (123)$$

In this case integrals B_2 and B_1 should be borrowed from Table 6 or Fig. 38.

The average/mean according to the expenditure/consumption speed of nucleus in the initial section will be located from the expression:

$$\left[\frac{u'_{cp}}{u_0}\right]_{2a}=\left[1-1{,}5\cdot\frac{aS}{R_0}\right]^2-2{,}56\cdot\frac{aS}{R_0}\cdot\int\limits_{1,17}^{-0,25}\left(\frac{u}{u_0}\right)^2\cdot d\varphi'+$$

$$+3{,}38\cdot\left[\frac{aS}{R_0}\right]^2\cdot\int\limits_{1,17}^{-0,25}\left(\frac{u}{u_0}\right)^2\cdot\varphi'\cdot d\varphi'.$$

Containing in this expression integrals it is not difficult to calculate according to Table 4:

$$\int\limits_{1,17}^{-0,25}\left(\frac{u}{u_0}\right)^2\cdot d\varphi'=-0.876;$$

$$\int\limits_{1,17}^{-0,25}\left(\frac{u}{u_0}\right)^2\cdot\varphi'\cdot d\varphi'=-0{,}544.$$

Hence average/mean according to the expenditure/consumption speed of the nucleus of constant mass in the initial section of the circular jet:

$$\left[\frac{u'_{cp}}{u_0}\right]_{2a}=1-0{,}76\cdot\frac{aS}{R_0}+0{,}47\cdot\left[\frac{aS}{R_0}\right]^2. \qquad (124)$$

In the transient section where occurs coupling of initial and basic sections, formulas (124) and (123) give the identical values of the average speed:

$$\left[\frac{u'_{cp}}{u_0}\right]_{2a}=\left[\frac{u_{cp}}{u_m}\right]=0{,}70.$$

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Fig. 40 depicts the curves of relative radii, kinetic energies and average/mean according to the expenditure/consumption speeds for the nucleus of the constant mass of circular jet.

Fig. 40 is constructed according to formulas (117)-(124).

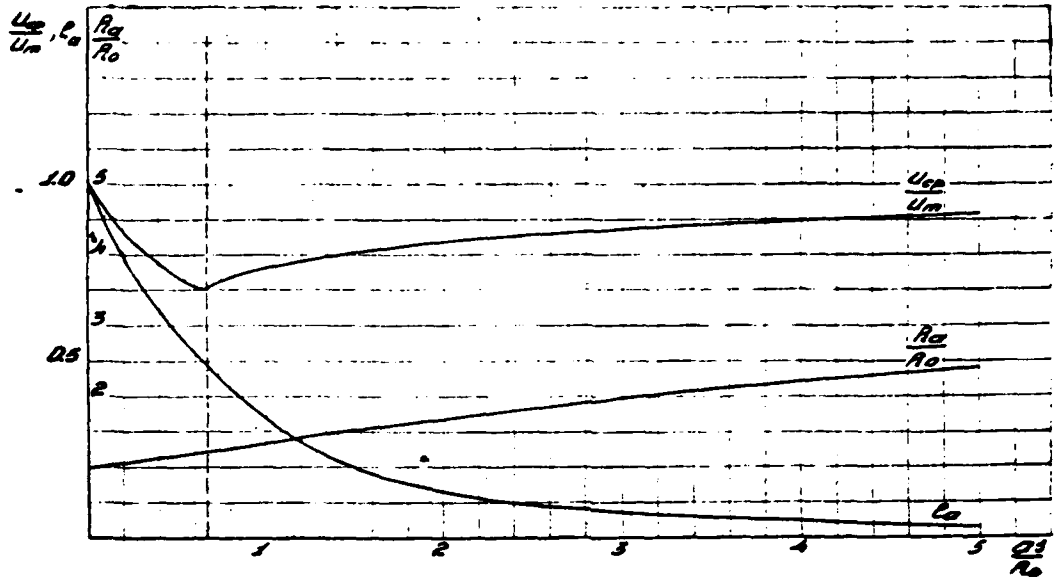

Fig. 40. Curves of a change in the relative radius, the kinetic energy and the average speed in the nucleus of the constant mass of circular jet.

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Part III.

EXAMPLES OF APPLICATION OF FREE BOUNDARIES THEORY.

1. Air curtain.

The production process of some industrial constructions (depot, hangars, garages, etc.) requires the periodic opening of enormous openings (gates) within the external walls.

In the winter time through these open openings are dug in the considerable masses of surrounding air, which call the intense cooling of working location. Fight with the cold by means of the heat installation under given conditions proves to be not profitable, since it requires colossal power expenditures.

Therefore usually instead of the thermal compensation of cooling, we try to overcome the the self-digging of surrounding air. One of the most rational methods of mechanical protection from the incidence/impingement of cold air into the location is air curtain. The operating principle of air curtain consists of the following. In the floor/sex directly before the open gates (Fig. 41) is had available the slit-shaped air duct from which escape/ensues the inclined (at angle α to the plane winch) slot jet of air with initial velocity u_0. The wind current of surrounding air, moving with speed v_0, encounters jet and the aim is to bend it to the side assignment. Under the action of the wind the jet is bent, and its curvilinear aerodynamic axis intersects with the plane winch. If intersection will lie/rest higher than the gates, then building will be completely shielded from the incidence/impingement in it of surrounding air. But if intersection is arranged below top winch, then the protective action of air curtain will be only partial, since in this case cold air will penetrate in the building through the clearance between the intersection and the top winch. The particle trajectory of the bent jet can be obtained by the geometric addition of the flows of the slot jet and wind. For this it is necessary to know each of the flows indicated individually. It is logical that the wind current on the basis of one or the other considerations it is possible to assign. However, the laws of the course of slot jet are studied in the first two parts of this work.

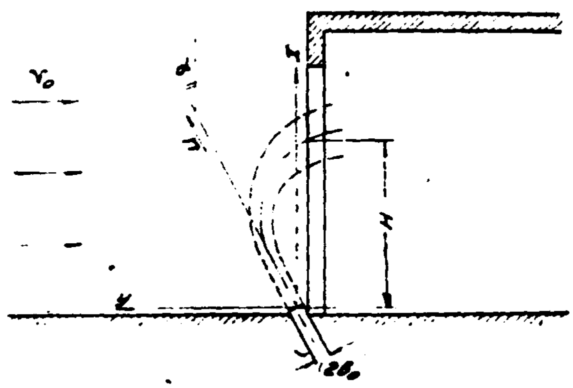

Fig. 41. Diagram of air curtain.

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Thus, are available all necessary prerequisites/premises for the solution of the problem about the air curtain. Comparatively large volume of this of problem forced us to make by its theme of separate investigation. In this example we will examine only approximate solution of the equation of the aerodynamic axis (axis of maximum speeds) of air curtain. For this purpose let us replace true jet with the fictitious jet whose particles move with average/mean in flow rate (u_{cp}) true jet velocity. Jet direct at angle α to the plane winch (Fig. 41).

Y axis let us arrange horizontally and direct against the wind. X axis let us combine with the plane winch and direct upward. Wind velocity we will consider time-constant and on the height/altitude [1].

FOOTNOTE [1]. The fundamental side of matter certainly will not change, if we take any other distribution of the speeds of wind. ENDFOOTNOTE.

The posed problem can be formulated as follows. To find the equation of bent axle of air curtain [y=f(x)], if are known wind velocity (v_0=const) and law of a charge in average/mean in the expenditure/consumption of speed initial slot jet $[u_{cp} = u_0 \varkappa(x)]$. Producing the geometric addition of the wind current and jet, we will obtain the following components of the speed of the air curtain:

$$\left.\begin{aligned} \frac{dy}{dt} &= V_y = u_{cp} \cdot \sin\alpha - v_0, \\ \frac{dx}{dt} &= V_x = u_{cp} \cdot \cos\alpha. \end{aligned}\right\} \qquad (125)$$

The differential equation of the axis of curtain will appear thus:

$$\frac{dy}{dx} = \frac{V_y}{V_x} = \operatorname{tg}\alpha - \frac{V_0}{\cos\alpha \cdot u_{cp}}. \qquad (126)$$

From the aerodynamic design of slot jet (§ 9) are known the following formulas of the average/mean according to the expenditure/consumption speed:

a) in initial section $\left(\frac{aS}{b_0} \leqslant 1{,}03\right)$:

$$\frac{u_{cp}}{u_0} = \frac{1}{1 + 0{,}43 \cdot \frac{aS}{b_0}};$$

b) in basic section $\left(\frac{aS}{b_0} \geqslant 1{,}03\right)$:

$$\frac{u_{cp}}{u_0} = \frac{u_{cp}}{u_m} \cdot \frac{u_m}{u_0} = 0{,}685 \cdot \frac{u_m}{u_0} = \frac{0{,}82}{\sqrt{\frac{aS}{b_0} + 0{,}41}}.$$

Here S - distance from the beginning of jet to its given section.

b_0 - half-width of initial jet cross-sectional area (half-width of exit slit of the air duct of curtain).

In this case when the plane of jet straddle x, it is necessary to keep in mind that:

$$S = \frac{x}{\cos\alpha}.$$

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Thus, in the initial section of the air curtain:

$$u_{cp} = \frac{u_0}{1 + 0{,}43 \cdot \frac{a \cdot x}{b_0 \cos\alpha}}. \qquad (127a)$$

In its basic section:

$$u_{cp} = \frac{0{,}82 \cdot u_0}{\sqrt{\frac{a \cdot x}{b_0 \cos\alpha} + 0{,}41}}. \qquad (127b)$$

The boundary between the basic and initial sections passes to the point:

$$\frac{a\cdot x}{b_0 \cos\alpha} = 1{,}03. \tag{127c}$$

Expressions (127a), (127b) and (127c) give the possibility to solve differential equation (126). Let us solve it first for the initial section of air curtain.

Here:

$$\frac{dy'}{dx} = \operatorname{tg}\alpha - \frac{v_0}{\cos\alpha\cdot u_0}\cdot\left(1 + 0{,}43\cdot\frac{a\cdot x}{b_0\cdot\cos\alpha}\right).$$

Let us introduce the designations:

$$\left.\begin{aligned} \bar{y} &= \frac{a\cdot y}{b_0\cdot\cos\alpha} \\ \bar{x} &= \frac{a\cdot x}{b_0\cdot\cos\alpha} \\ \bar{v} &= \frac{v_0}{u_0\cos\alpha} \end{aligned}\right\}.$$

Then

$$\frac{dy'}{d\bar{x}} = \operatorname{tg}\alpha - \bar{v} - 0{,}43\cdot\bar{v}\cdot\bar{x}. \tag{128}$$

Whence

$$\bar{y}' = \operatorname{tg}\alpha\cdot\bar{x} - \bar{v}\cdot\bar{x} - 0{,}215\cdot\bar{v}\cdot\bar{x}^2 + c_1. \tag{129}$$

It is not difficult to surmise, that with $\bar{x}$=0:

$$\bar{y}' = 0, \text{ i.e. } c_1 = 0.$$

Hence we obtain the final equation of the axis of the initial section of the air curtain:

$$\bar{y}' = \operatorname{tg}\alpha\cdot\bar{x} - \bar{v}\cdot\bar{x} - 0{,}215\cdot\bar{v}\cdot\bar{x}^2. \tag{130}$$

At the end of initial section ($\bar{x}$=1.03) we obtain the following ordinate of air curtain:

$$y_0' = 1{,}03 \cdot \operatorname{tg} \alpha - 1{,}26 \cdot \bar{v}_1. \qquad (131)$$

Let us find now the equation of the axis of the basic section of air curtain. In this case:

$$\frac{dy}{dx} = \operatorname{tg} \alpha - 1{,}22 \cdot \bar{v} \cdot \sqrt{x + 0{,}41} \cdot \qquad (132)$$

Whence

$$y = \operatorname{tg} \alpha \cdot x - 0{,}81 \cdot v\,(x + 0{,}41)^{\frac{3}{2}} + c_2. \qquad (133)$$

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In the beginning of basic section ($\bar{x}$=1.03) and at the end of the initial section the ordinates of curtain must coincide:

$$y_0 = y_0'.$$

However, with x=1.03:

$$y_0 = 1{,}03 \cdot \operatorname{tg} \alpha - 1{,}41 \cdot v + c_2;$$
$$y_0' = 1{,}03 \cdot \operatorname{tg} \alpha - 1{,}26 \cdot \bar{v}.$$

Therefore

$$c_2 = 0{,}15 \cdot v.$$

Thus, the equation of the axis of the basic section of air curtain takes the following form:

$$y = \operatorname{tg} \alpha \cdot x - 0{,}81 \cdot v \cdot (x + 0{,}41)^{\frac{3}{2}} + 0{,}15 \cdot v. \qquad (134)$$

However, during finding of the trajectory of the axis of air curtain should be first determined the crdinate of transient section ($y_0=y_0'$, and then in region $x<1.03$ perform calculations according to the formula (130), in the region $x>1.03$ - according to formula (133).

Is recommended the fcllcwing crder of the determination of the trajectory of the axis of the air curtain:

1. To select initial values - a_0, v_0, u_0, b_0, a and to calculate $v=(v_0/u_0 \cos \alpha)$;

2. To assign different values of x and to find the appropriate values of $x=(a \cdot x/b_0 \cos \alpha)$;

3. From equations (130) and (133) to find values of $y=f(x)$;

4. To calculate the apprcpriate values of $y=(y \cdot b_0 \cdot \cos \alpha/a)$;

5. To construct the unkncwn curve $y=f(x)$.

It is necessary to ncte that fcr designing the air curtain there is the greatest interest in a questicn about its range. Mcreover by range we understand distance frcm the bottcm winch to the intersection of the axis of curtain with the plane winch. Ordinate of intersection: $y_* = 0.$

Therefore, the range of curtair is determined by the condition:

$$0 = \operatorname{tg}\alpha \cdot x_{\kappa} - 0{,}81 \cdot \bar{v} \cdot (x_{\kappa} + 0{,}41)^{2} + 0{,}15 \cdot \bar{v}. \qquad (135)$$

Attempting to simplify linings/calculations, let us disregard/neglect the low values (0.41 and 0.15 $\bar{v}$), then:

$$x_{\kappa} = 1{,}5 \cdot \frac{\operatorname{tg}^2\alpha}{\bar{v}^2} = 1{,}5 \cdot \left(\frac{u_0}{v_0}\right)^2 \cdot \sin^2\alpha. \qquad (136)$$

Let us designate distance from the bottom of winch to the intersection of curtain with the plane winch by letter H. Therefore the final equation of the range of the air curtain will take the following form:

$$H = \frac{x_{\kappa} \cdot b_0 \cdot \cos\alpha}{a}$$

or, that the same

$$\frac{H}{b_0} = \frac{1{,}5}{a} \cdot \left(\frac{u_0}{v_0}\right)^2 \cdot \sin^2\alpha \cdot \cos\alpha. \qquad (137)$$

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Let us find now such a value cf the angle of the jet inclination (α), with which air curtain will pcssess maximum range. It is cbvious that this value of angle α ccrrespcnds tc the condition:

$$\frac{dH}{d\alpha}=0.$$

Whence

$$2\cdot\cos^2\alpha-\sin^2\alpha=0.$$

$$\operatorname{tg}\alpha=1{,}41.$$

i.e.

Thus, the angle of the maximum range of the air curtain:

$$\alpha_{opt}=\operatorname{arctg}(1{,}41)=54^\circ 40'. \qquad (138)$$

If we substitute this value of angle α intc expression (137), then will be obtained the formula cf the maximum range of the curtain:

$$\frac{H_{max}}{b_0}=\frac{0{,}58}{a}\cdot\left(\frac{u_0}{v_0}\right)^2. \qquad (139)$$

At assigned wind velocity it is pcssible tc attain an increase in the range of curtain either due tc an increase in the width of jet (b_0), or due to an increase in the discharge velocity (u_0). Let us note that from an energy point of view it is profitable tc combine the large width of curtain with the low speed of discharge. About this testifies equation (139), in which velocity (u_0) is contained squared, and width (b_0) - to the first degree.

In conclusion it is necessary to indicate the fact that for the first time the problem about the air curtain was solved by eng. V. V. Baturin [1].

FOOTNOTE [1]. See V. V. Baturin and I. A. Shapelev. Air of curtains, the journal "heating and the ventilation" of No 5, 1936. ENDFOOTNOTE.

With the solution of this problem V. V. Baturin used the formulas of average/mean by the area jet velocity. It was subsequently established/installed, which more substantiated to use average/mean according to the flow rate velocity, thanks to which was required to rework the calculation of air curtain. By this is explained the fact that the results of our solution differ from the results of deciding V. V. Baturin.

2. Resistance of the labyrinth seal of blower.

The internal pressure of air blower usually differs from atmospheric. Due to this occurs constant air leakage through the clearance between wheel and jacket of blower. For the reduction of the sizes/dimensions of leakage to clearance they attempt to shape in the form of the labyrinth which creates resistance to motion of air and it impedes the formation/education of larger flow rate through the clearance. The diagram of the cell of labyrinth is depicted in Fig. 42.

Complete labyrinth seal consists of the series/row of the consecutive cells.

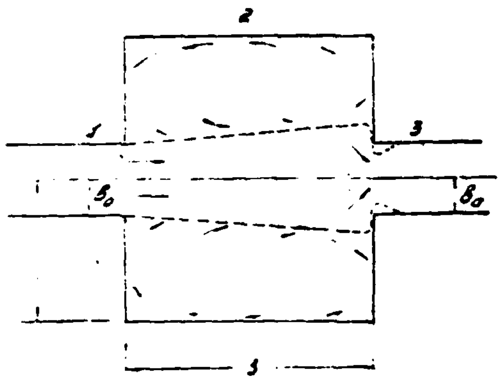

Fig. 4.2. The aerodynamic configuration of flow in the cell of labyrinth.

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It is logical that resistance of labyrinth is equal to the sum of resistances of separate cells. Therefore for the analysis of resistance of labyrinth it suffices to examine resistance of one cell. The physical essence of flow in the cell of labyrinth is reduced to the following: from slot 1 escape/ensue into the chamber/camera 2 airstreams with a a velocity of of u_0. Being spread in the chamber/camera, jet is expanded and mixes to itself the particle of the surrounding stagnant air. At the end of the chamber/camera from the jet is isolated the nucleus of constant mass, which gathering, flows into slot 3. The connected masses of air scale from the nucleus and, undergoing backward motion along the chamber walls, again they are mixed to the active jet. Conceal by form, free flow (Freistrahl) in the cell of labyrinth is characteristic fact that the nucleus of constant mass passes through the cell while the apparent additional mass it forms around the jet locked circulation of air particles. Resistance of cell consists of energy losses of the nucleus of constant mass. The latter are composed of two parts:

1) difference in the supplies of energy of the nucleus of constant mass in the beginning and at the end of the cell;

2) energy losses of the sudden compression of the nucleus of constant mass with its inflow into slot 3.

Let us examine each of the composite/compound component parts of resistance of labyrinth separately. For this the circular form of labyrinth let us replace with flat/plane [1].

FOOTNOTE [1]. The width of labyrinth ring is always so small in comparison with its diameter that without any doubts the annular slot can be replaced with flat/plane. ENDFOOTNOTE.

In this case we will deal concerning the studied above flat/plane free jet. Relative energy losses in the initial section of the nucleus of the constant mass of slot jet are equal (see formula 106);

$$\Delta e'_{u} = 1 - e'_{u} = 0.275 \cdot \frac{aS}{b_{u}}. \qquad (140)$$

In this case: S - length of the cell (chamber/camera) of labyrinth; b_0 - the half-width of slots 1 and 3; Δe_a - energy loss in the portions of the kinetic energy of jet in slot 1.

In the region of basic section the decrease of energy of the nucleus of constant mass comprises (see formula 105):

$$\Delta e_a = 1 - e_a = 1 - \frac{1{,}73 \cdot A_3}{\sqrt{\frac{aS}{b_0} + 0{,}41}}. \qquad (141)$$

Coefficient $A_3 = f_1(\varphi_a)$ is determined from table 5 or Fig. 33, where:

$$\varphi_a = f_2(A_1).$$

In this case, accordingly formula (101):

$$A_1 = \frac{0{,}833}{\sqrt{\frac{aS}{b_0} + 0{,}41}}.$$

If the length of labyrinth cell does not exceed the length of the initial section of jet ($aS/b_0 < 1.03$), then should be used formula (140). Otherwise ($aS/b_0 \geq 1.03$) it is necessary to resort to formula (141), the order of the use of which is in detail presented in § 9 of this work.

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The second part of resistance of labyrinth (input resistance into slot 3) can be determined according to the known from the hydraulics empirical formula of sudden jet contraction:

$$\Delta e_{s}=0,5\cdot\left(1-\frac{u_{cp}}{u_0}\right)^2. \qquad (142)$$

In this formula: Δe_s - resistance of the sudden contraction of flow, expressed in the portions of the kinetic energy in the narrow part (in slot 3); u_{cp} - the average speed of flow before contraction; u_0 - the average speed of flow after contraction. Value $\frac{u_{cp}}{u_0}$ is determined in the theory of jet. Let us recall that the discussion deals with relative value of average according to the flow rate of the velocity in the nucleus of constant mass, which in the initial section is determined by formula (108):

$$\frac{u_{cp}}{u_0}=1-0,16\cdot\frac{aS}{b_0}, \qquad (143)$$

and in the basic section - by formula (107):

$$\frac{u_{cp}}{u_0}=\frac{u_{cp}}{u_m}\cdot\frac{u_m}{u_0}=\frac{1,2}{\sqrt{\frac{aS}{b_0}+0,41}}\cdot\frac{A_2}{A_1}.$$

Here:

$$A_1=\frac{0,833}{\sqrt{\frac{aS}{b_0}+0,41}};$$

$$A_2=f_3(\varphi_a);$$

$$\varphi_a=f_2(A_1).$$

Therefore in the basic section:

$$\frac{u_{cp}}{u_0}=1,44\cdot A_2. \qquad (144)$$

The complete coefficient of resistance of the cell of labyrinth, which expresses energy losses in the portions of the kinetic energy in slot 1, is equal to the sum of coefficients $\Delta e_н$ and $\Delta e_в$:

$$\xi = \Delta e_н + \Delta e_в. \qquad (145)$$

In the limits of the initial section where $aS/b_0 \leqslant 1.03$, drag coefficient comprises [see formulas (140), (142), (143)]:

$$\xi' = \Delta e'_н + \Delta e'_в = 0{,}275 \cdot \frac{aS}{b_0} + 0{,}0128 \cdot \left[\frac{aS}{b_0}\right]^2. \qquad (146)$$

In the basic section when $aS/b_0 \geqslant 1.03$, drag coefficient is equal to:

$$\xi = 1 - \frac{1{,}73 \cdot A_в}{\sqrt{\frac{aS}{b_0} - 0{,}41}} + 0{,}5\,[1 - 1{,}44 \cdot A_в]^2. \qquad (147)$$

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If the length of the cell of labyrinth is equal to the length of initial section ($aS/b_0 = 1.03$) of slot jet, then in the equal measure are suitable both of formulas.

In this case:

$$\xi' = \xi = 0{,}3.$$

In order to simplify the use of formula (147), let us recall the rational order of the calculations:

1) From condition $A_1 = \frac{0{,}833}{\sqrt{\frac{aS}{b_0} + 0{,}41}}$ find value A_1[1].

FOOTNOTE [1]. Values A select in accordance with the data of § 6 (just as during the calculation of slot jet). ENDFOOTNOTE.

2) On Table 5 or Fig. 34 they determine $\varphi_a = f_2(A_1)$, and also $A_2 = f_3(\varphi_a)$ and $A_3 = f_1(\varphi_a)$.

3) Substituting values A_2 and A_3 into formula (144), they calculate the drag coefficient of labyrinth.

Fig. 43 depicts the dependence of resistance of the cell of labyrinth on its relative length:

$$\xi = f\left(\frac{aS}{b_0}\right)$$

Very closely to the curve Fig. 43 are arranged/located the experimental points, obtained by K. V. Chebysheva in the ventilator laboratory of TsAGI [2].

FOOTNOTE [2]. K. V. Chebysheva. Investigation of the model of labyrinth seal. It is printed in tech. notes of TsAGI. ENDFOOTNOTE.

It is interesting to focus attention on the fact that during working/treatment of experiments, we accepted a=0.11, i.e., introduced no corrections for jet deflection in the labyrinth from the jet in the unlimited space. As is evident, in the experiences of TsAGI the cell of labyrinth was roomy [3].

FOOTNOTE [3]. In the experiences of TsAGI the half-width of cell was equal to 10 half-widths of slot 1, and the length of cell was varied in the limits =(0-23)bo. ENDFOOTNOTE.

Let us note that with a small half-width (l) of the cell when it is less than the half-width of free jet, formula (146)-(147) resistances of labyrinth seal are unsuitable. In this case the jet will fill all the section of cell, and energy losses will consist of losses by shock during the sudden expansion of jet and during its subsequent contraction:

$$\zeta_{n} = 1{,}5 \cdot \left[1 - \frac{u_{*}}{u_{0}}\right]^{2}. \qquad (148)$$

Here u_{*} - velocity in the cell with the continuous filling of its section with airflow.

Thus, before calculating labyrinth, should be determined relative value of the half-width of the free jet

$$\frac{b_{\text{гр}}}{b_{0}} = 2{,}4 \cdot \frac{aS}{b_{0}} + 1$$

and then, if it seems that

$$\frac{b_{\text{гр}}}{b_{0}} > \frac{l}{b_{0}},$$

to resort to formula (148).

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But if it is obtained:

$$\frac{b_{\text{гр}}}{b_{0}} \leq \frac{l}{b_{0}},$$

then should be utilized formulas (146) and (147).

3. External resistance of the corridor bank of tubes.

Air flow in the corridor beam of ducts (Fig. 44) has much in common with the dismantled/selected above flow at the labyrinth.

From slots 1 in first run of pipes escape/ensue the air filaments and, being expanded, they are spread in the between-row space. Here to the basic nucleus of stream are mixed apparent additional masses from the shadow regions [1].

FOOTNOTE [1]. Shadow we call the regions, situated after the tubes of bundle. ENDFOOTNOTE.

Flowing to the slots of second run of pipes, streams are split. In this case basic nucleus is passed in the second series/row of tubes, and apparent additional mass forms the locked air circulation in the shadow regions. The schematic of flow in the second and subsequent between-row spaces almost in no way differs from the diagram of the first between-row space. Deviation consists only in the fact that the turbulence of flow after each subsequent series/row is somewhat more than after preceding/previous. Is explained this by the intense agitation of free streams in the between-row spaces. The ducts of corridor bundle are usually rectilinear; therefore the described free jets can be considered flat/plane.

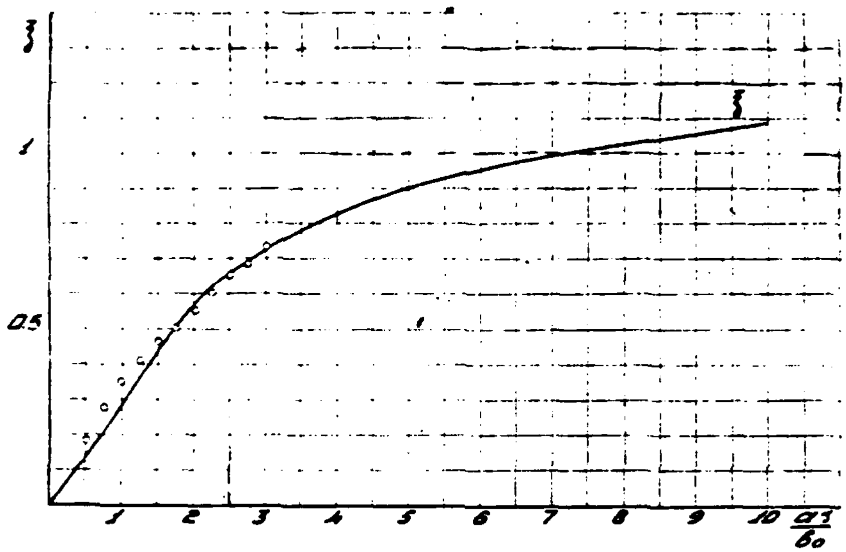

Fig. 43. Dependence of the drag coefficient of the cell of labyrinth on its relative length.

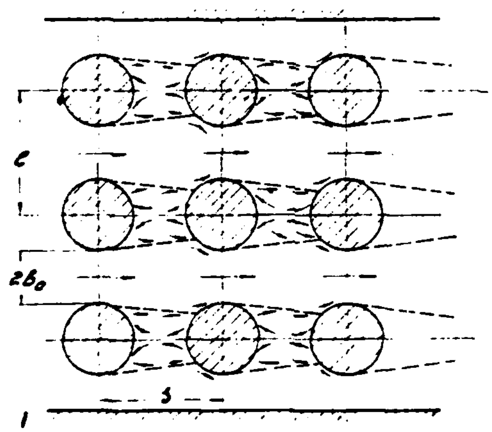

Fig. 44. Flow diagram in corridor bank of tubes.

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The resistance of the between-row space of corridor bundle must be defined just as the resistance of the cell of labyrinth, with the only the difference, that because of the cylindrical form of tubes, the compression of the nucleus of constant mass at the end of the between-row region (before the subsequent series/row of tubes) will be smooth and resistance from jet contraction virtually will fall.

Thus, the drag coefficient of between-row space is equal to relative energy loss in the nucleus of the constant mass of slot jet at the length of the between-row space:

$$\xi = 1 - e_a. \quad (149)$$

When the length of between-row space does not exceed the length of the initial section of the jet [see formula (106)] we will have:

$$\xi' = 0{,}275 \cdot \frac{aS}{b_0}. \quad (150)$$

Here S - longitudinal distance between the neighboring series/rows; b_0 - half-width of the transverse between-row space is longer than the initial section, then [see formula (105)]:

$$\xi = 1 - \frac{1{,}73 \cdot A_3}{\sqrt{\frac{aS}{b_0} + 0{,}41}}. \quad (151)$$

Moreover value A_3 is determined as follows:

1) Through formula $A_1 = \frac{0{,}833}{\sqrt{\frac{aS}{b_0} + 0{,}41}}$, we find A_1.

2) On Table 5 or Fig. 33, we find $\varphi_a = f_2(A_1)$, and then $A_3 = f_1(\varphi_a)$.

We emphasize that the values of the coefficient of jet structure a in different between-row spaces are dissimilar. In the first space a_1 it corresponds to usual slot jet ($a_1=a_0\approx 0.11$).

In the second space $a_2>a_1$, since here flow is additionally created turbulence due to the free flow in the first space.

In the subsequent spaces:

$$a_3>a_2,$$
$$a_4>a_3,$$
$$a_5>a_4 \text{ and etc.}$$

However, already beginning from the third space, flow is so created turbulence that its subsequent agitation hardly will be able to increase substantially value of a. In other words, it is possible to expect that:

$$a_3=a_4=a_5=\ldots \text{ and etc.}$$

One should also indicate the fact that values a_2, a_3, a_4 ... depend not only on the agitation of flow in the preceding/previous between-row spaces, but also on its deturbulization during the compression of jet in the clearances of each run of pipes [1].

FOOTNOTE [1]. See G. N. Abramovich. The "principles of the aerodynamic design of collector/receptacle". Transactions of TsAGI iss. 231.

Moscow 1935. ENDFOOTNOTE.

Quantitative estimate of the magnitude a_2, a_3, a_4 ... extremely hinders by the absence at our disposal of the corresponding experimental material.

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Let us attempt to nevertheless give the tentative analysis of the resistance of the corridor bank of tubes based on one particular example in which:

1) the longitudinal pitch (between the series/rows) of the bundle:

$$S=4\cdot b_0,$$

where b_0 - half-width of transverse clearance;

2) transverse pitch (between the ducts of one series/row):

$$l=4\cdot b_0;$$

3) the coefficient of the structure of slot jet - a_0=0.11.

The analysis of experiments of Syrkin shows that in the jet after the lattice of ducts coefficient a, approximately/exemplarily, is 1.5 times more than usual. Therefore for calculating all between-row spaces of bundle we accept:

$$a = 1,5 \cdot a_0 \cong 0,17.$$

Thus, the relative longitudinal pitch:

$$\frac{aS}{b_0} = 0,68 < 1,03.$$

Drag coefficient of one between-row space:

$$\xi' = 0,275 \cdot 0,68 = 0,187.$$

In all in by five-dowlas bundle are four between-row spaces. Their total resistance:

$$4 \cdot \xi' = 0,75.$$

Furthermore, on leaving from the latter/last series/row flow is expanded and fills all the section of channel. Therefore in the latter/last series/row of bundle are observed the energy losses to the shock:

$$\xi_{yg} = \left(1 - \frac{2b_0}{l}\right)^2 = 0,25.$$

Thus, the coefficient of total drag of the beam:

$$\Sigma\xi = 4 \cdot \xi' + 0,25 = 1,0.$$

According to experiments of Reiher [1] for the corridor bundle of the selected configurations it is obtained:

$$\Sigma\xi = 1,0 - 1,1.$$

FOOTNOTE [1]. H Reiher. Wärmeübergang von strömender Luft an Rohre und Röhrenbündel im kreuzstrom. Forschungsarbeiten auf dem Gebiete des Ingenieurwesens herausgegeben vom V. D. I. Heft 269. ENDFOOTNOTE.

As we see, our rough estimate can be considered successful. Let us note in conclusion that without having available a sufficient experimental material, we could not the aerodynamic analysis of the bank of tubes bring to the end/lead, and therefore presented should be examined only as the attempt to develop the physically substantiated calculation method.

4. Warm and cold airstreams.

Frequently with the ventilation and the hot-air heating of construction constructions they resort to the forcing into the locations of cold or warm airstreams. The trajectories of the air jets, which escape into the air medium of another density, cannot be rectilinear.

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Cold jet due to surplus density attempts to be dropped/omitted down. On the contrary, warm jet is displaced by surrounding air upward. Thus, the laws of the rectilinear propagation of the free flow are not completely applicable to the warm and cold airstreams, since in this case appear the bending flow gravitational forces. It is necessary to note that the problem about the curvilinear jet was as early as 1934 set by V. V. Gaturin [1] which it solved it on the basis of the empirical data about the rectilinear jet.

FOOTNOTE [1]. See V. V. Baturin and I. A. Shepelev. Approximate determination of the trajectory of air flow. "heating and the ventilation" of No 9, 1934. ENDFOOTNOTE.

In this example, with the aid of the theory of jet, we will derive the approximate equation of the aerodynamic axis of circular jet, which is bent by gravitational forces [2].

FOOTNOTE [2]. Just as in the case of air curtain, we replace the problem about the jet of the approximated by the problem about the aerodynamic axis jet. ENDFOOTNOTE.

For this let us assume that:

1) Entire flow mass is moved with the average/mean according to the flow rate velocity.

2) Initial jet direction is horizontal. In this case the horizontal component of the velocity will be equal to average/mean according to the flow rate of the velocity of rectilinear jet (u_{cp}). However, vertical component of velocity (v_{cp}) will be determined by a difference in the specific gravity/weights in the jet and in the

environment.

3) By the centrifugal forces which are caused by the curvilinearity of jet, it is disregarded.

Problem we will solve in the rectilinear coordinate system. Y axis it is directed vertically upward. X axis - it is horizontal, with the flow.

The origin of coordinates consistent with the center of initial jet cross-sectional area (Fig. 45).

Let us select the arbitrary element/cell of jet dx with abscissa x. On this element/cell will act the lift, equal to the product of its volume to a difference in the specific gravity/weights:

$$P=(\gamma_{ata}-\gamma_{cp})\cdot F\cdot dx.$$

Here γ_{ata} - specific gravity/weight of surrounding air; γ_{cp} - average specific gravity/weight of the element/cell of jet; F - sectional area of jet; dx - thickness of the element/cell of jet.

Force P will cause the vertical acceleration:

$$\frac{dv_{cp}}{dt}=\frac{P}{M},$$

where $M=\frac{\gamma_{cp}}{g}\cdot F\cdot dx$ - mass of the selected element/cell of jet.

Thus, the vertical acceleration cf jet is equal to:

$$\frac{dv_{cp}}{dt}=g\cdot\left(\frac{\gamma_{a\text{т}}}{\gamma_{cp}}-1\right).$$

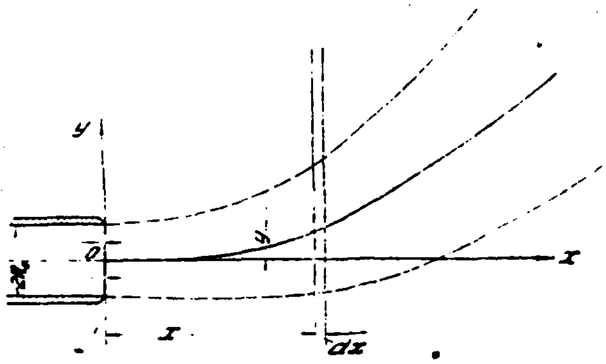

Fig. 45. Bending of jet by gravitational forces.

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Pressures in the jet and cut of the jet are identical; therefore specific gravity/weights are inversely proportional to the absolute temperatures:

$$\frac{\gamma_{ata}}{\gamma_{cp}} = \frac{T_{cp}}{T_{ata}},$$

whence

$$\frac{dv_{cp}}{dt} = g \cdot \frac{\Delta T_{cp}}{T_{ata}}. \qquad (152)$$

In this expression: g - acceleration of gravity; $\Delta T_{cp} = T_{cp} - T_{ata}$ - excess temperature of jet in section x; T_{ata} - absolute temperature of surrounding air.

Horizontal jet velocity:

$$\frac{dx}{dt} = u_{cp}.$$

therefore

$$dv_{cp} = g \cdot \frac{\Delta T_{cp}}{T_{ata}} \cdot dt = g \cdot \frac{\Delta T_{cp}}{T_{ata}} \cdot \frac{dx}{u_{cp}}.$$

Introducing into this expressicn the initial velocity of jet (u_0) and initial excess temperature ($\Delta T_0 = T_0 - T_{ata}$), we obtain:

$$dv_{cp} = g \cdot \frac{\Delta T_{cp}}{\Delta T_0} \cdot \frac{\Delta T_0}{T_{ata}} \cdot \frac{u_0}{u_{cp}} \cdot \frac{dx}{u_0}.$$

In § 7 it is shown that the fields cf the excess temperatures of free jet are similar the velccity fields:

$$\frac{\Delta T_{cp}}{\Delta T_0} = \frac{u_{cp}}{u_0}.$$

Thus, we have:

$$dv_{cp} = g \cdot \frac{\Delta T_0}{T_{ata}} \cdot \frac{dx}{u_0};$$

$$v_{cp} = g \cdot \frac{\Delta T_0}{T_{ata}} \cdot \frac{x}{u_0} + c_1.$$

In the beginning of jet when $x=0: v_{cp}=0$, i.e. $c_1=0$, thanks to which the formula of the average/mean vertical velocity appears as follows:

$$\frac{v_{cp}}{u_0} = \frac{g}{u_0^2} \cdot \frac{\Delta T_0}{T_{ata}} \cdot x. \qquad (153)$$

Attempting to simplify the fcllcwing linings/calculations, let us introduce the relative coordinates:

$$x = \frac{a \cdot x}{R_0};$$

$$y = \frac{a \cdot y}{R_0}.$$

Then the vertical velocity:

$$\frac{v_{cp}}{u_0} = \frac{1}{a} \cdot \frac{g \cdot R_0}{u_0^2} \cdot \frac{\Delta T_0}{T_{ata}} \cdot x. \qquad (154)$$

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From the point of view of average/mean according to the flow rate horizontal speed, the jet one should break in the initial and basic sections.

In the initial section [see formula (106a)]:

$$\frac{u'_{cp}}{u_0}=\frac{1}{1+0,76\cdot\bar{x}+1,32\cdot\bar{x}^2}. \qquad (155)$$

In the basic section:

$$\frac{u_{cp}}{u_0}=\frac{0,45}{\bar{x}+0,29}. \qquad (156)$$

As is known,

$$\frac{dy}{dx}=\frac{v_{cp}}{u_{cp}}. \qquad (157)$$

Therefore, having available formulas (154), (155) and (156), it is possible to compose the differential equation of the aerodynamic axis of free jet, which is bent by gravitational forces. Let us comprise and will solve this equation for the initial section of the jet:

$$\frac{d\bar{y}'}{d\bar{x}}=\frac{1}{a}\cdot\frac{g\cdot R_0}{u_0^2}\cdot\frac{\Delta T_0}{T_{ata}}\cdot\bar{x}\cdot(1-0,76\cdot\bar{x}-1,32\cdot\bar{x}^2);$$

$$\bar{y}=\frac{1}{a}\cdot\frac{g\cdot R_0}{u_0^2}\cdot\frac{\Delta T_0}{T_{ata}}\cdot\left[\int\bar{x}\cdot d\bar{x}+\int 0,76\cdot\bar{x}^2\cdot d\bar{x}+\int 1,32\cdot\bar{x}^3\cdot d\bar{x}+c_2\right].$$

After producing the necessary linings/calculations, we will obtain:

$$\bar{y}'=k\cdot\bar{x}^2\cdot[0,5-0,25\cdot\bar{x}+0,3\cdot\bar{x}^2+c_2],$$

where

$$k=\frac{1}{a}\cdot\frac{g\cdot R_0}{u_0^2}\cdot\frac{\Delta T_0}{T_{ata}}.$$

However with $\bar{x}$=0:

$$\bar{y}'=0, \text{ i. e. } c_2=0.$$

Thus, the equation of the axis of the initial section of curvilinear circular jet takes the following form:

$$\bar{y}'=k\cdot\bar{x}^2\cdot[0,5+0,25\cdot\bar{x}-0,3\cdot\bar{x}^2]. \qquad (158)$$

At the end of the initial section where $\bar{x}_0=a\cdot x_0/R_0=0.67$:

$$\bar{y}_0=0,36\cdot k.$$

Let us comprise and will solve row equation for the basic section of the jet:

$$\frac{dy}{dx}=2,22\cdot k\cdot x\cdot(x-0,29);$$

$$y=k\cdot x^2\cdot[0,32+0,67\cdot x+c_3].$$

Integration constant c_3 can be determined from that condition that in transient jet cross-sectional area ($\bar{x}_0=0.67$):

$$y_0=\bar{y}_0=0,36\cdot k,$$

whence

$$c_3=0.$$

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The equation of the axis of the basic section of the bent jet takes the following form:

$$y=k\cdot x^2\cdot[0,32+0,74\cdot x]. \tag{159}$$

Let us recall that in equation (159) just as in equation (158), is accepted the designation:

$$k=\frac{1}{a}\cdot\frac{g\cdot R_0}{u_0^2}\cdot\frac{\Delta T_0}{T_{\text{ок}}}. \tag{160}$$

The examined by us case of curvilinear jet is determined by interaction of gravitational forces and forces of inertia of the moving/driving jet. From the theory of aerodynamic similarity it is known that in this case air flcw must be characterized by the special similarity criterion which is called the criterion of Froude:

$$\Phi = \frac{u^2}{g \cdot l}.$$

Here g - acceleration of gravity; l - linear dimension; u^2 - square of velocity.

Scrutinizing into equality (160), we and actually detect in it as the factcr Froude's criteric r:

$$\Phi_0 = \frac{u_0^2}{g \cdot R_0}, \quad (161)$$

in other words,

$$k = \frac{1}{a \cdot \Phi_0} \cdot \frac{\Delta T_0}{T_{ata}}. \quad (162)$$

In conclusion we recommend the following order of the determination of the trajectcry cf the warm or cold circular jet:

1) knowing initial velccity (u_0), initial temperature (T_0), temperature of surrounding air (T_{ata}) and coefficient of jet structure (a), we compute Froude's criterion:

$$\Phi_0 = \frac{u_0^2}{g \cdot R_0}$$

and the parameters:

$$\frac{\Delta T_0}{T_{ata}} = \frac{T_0 - T_{ata}}{T_{ata}};$$

$$k = \frac{1}{a \cdot \Phi_0} \cdot \frac{\Delta T_0}{T_{ata}}.$$

2) We find abscissa and crdinate of transient jet cross-sectional area:

$$x_0 = \bar{x}_0 \cdot \frac{R_0}{a} = 0{,}67 \cdot \frac{R_0}{a};$$

$$y_0 = \bar{y}_0 \cdot \frac{R_0}{a} = 0{,}36 \cdot \frac{k \cdot R_0}{a}.$$

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3. We are assigned by different values of x and compute values of $\bar{x}=a \cdot x/R_0$. Further, through equations (158) and (159) we find the appropriate values of $\bar{y}$. In this case, if $\bar{x} \leq 0.67$, then is utilized equation (158), but if $\bar{x} \geq 0.67$ - equation (159).

4. Finally, from condition $\bar{y}=a \cdot y/R_0$ we obtain unknown values y and we construct curvilinear axis of jet:

$$y = f(x).$$

For the cold jet when $\Delta T_0<0$, we obtain the negative values of ordinates - jet will be bent downward. At the same time for warm jet, in which $\Delta T_0>0$, will be discovered the bending of jet upwards.

* * *

The use/application of theory and aerodynamic design of free jet is by far not contained by the dismantled/selected above examples. With the aid of the theory of jet it is possible to conduct the calculations of the flames of combustion, the calculations of the effectiveness of the through ventilation of industrial shops and many others. Without having the capability in the limits of this work it will stop at all these problems, it will hope for the fact that they will serve as the themes of further investigations.

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Supplement.

Calculation of the expanding turbulent flow [1].

FOOTNOTE [1]. W. Tollmien "Berechnung turbulenter Ausbreitungsvorgänge" ZAMM Bd 6. Heft. 6. 1926. ENDFOOTNOTE.

W. Tollmien.

II. Expansion of jet as two-dimensional problem [2].

FOOTNOTE [2]. Is given below the translation/conversion of the 2nd and 3rd chapters of the article of Tollmien, used by the author of this work. The translation/conversion of the 1st chapter is given in appendix I to the investigation: G. N. Abramovich. Aerodynamics of flow in open wind-tunnel test section. H. 1, the transactions of TsAGI, iss. 223, 1935. ENDFOOTNOTE.

From the wall through the narrow slot which during calculations can be replaced with line, ensues/escapes/flows out the airstream and is mixed with the surrounding stagnant air.

If we consider that in the jet rules the pressure, equal to external, then utilizing a theorem of momentum, it is possible to produce separation of variables. As the variable/alternating let us again use x and η.

As a result of the pressure constancy impulse/momentum/pulse in the direction of x axis must remain constant/invariable:

$$\int_{-x}^{+x} u^2 \cdot dy = \text{const.}$$

Assuming/setting

$$u = \varphi(x)\cdot f(\eta).$$

we will obtain

$$\varphi^2(x)\cdot x \int_{-x}^{+x} f^2(\eta)\cdot d\eta = \text{const.}$$

Consequently

$$\varphi(x) = \frac{1}{\sqrt{x}};$$

$$u = \frac{1}{\sqrt{x}}\cdot f(\eta) \qquad (13a)$$

$$\psi = \int \frac{1}{\sqrt{x}}\cdot f(\eta)\cdot d\eta = \sqrt{x}\cdot \int f(\eta)\cdot d\eta = \sqrt{x}\cdot F(\eta). \qquad (13b)$$

$$v = -\frac{1}{2\cdot\sqrt{x}}\cdot F(\eta) + \frac{1}{\sqrt{x}}\cdot F'(\eta)\cdot \eta. \qquad (13c)$$

Now, as in § 1, it is possible to compose the differential equation which in this case also is integrated. The same intermediate integral it is possible to obtain directly from the theorem of momentum. If we conduct control surface in the manner that this is done in Fig. 8, then through its lower boundary will be transferred impulse/momentum/pulse $\rho \cdot u \cdot v$; from opposite side will occur a change in the impulse/momentum/pulse:

$$\rho \cdot \frac{\partial}{\partial x} \int_x^y u^2 \cdot dy.$$

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As the external force will act turbulent shearing stress

$$\tau = \rho \cdot c^2 \cdot x^2 \cdot \frac{\partial u}{\partial y} \cdot \frac{\partial u}{\partial y}.$$

Thus, appears the following relationship/ratio:

$$u \cdot v + \frac{\partial}{\partial x} \int_x^y u^2 \cdot dy = \frac{\tau}{\rho}.$$

Hence we obtain equation for $F(\eta)$:

$$2 \cdot c^2 \cdot [F'']^2 = F \cdot F'. \tag{14}$$

(It usefully for positive values η; the distribution of the velocities in the region of negative ones η is obtained as the mirror image).

Introducing the appropriate scale for η, simplify the differential equation:

$$[F'']^2 = F \cdot F'. \tag{14a}$$

The order of this differential equation can be lowered, if to introduce new dependant variable z=ln F, otherwise F=e³. Then we obtain [z''+(z')²] ²=z' and, after assuming z'=Z, let us arrive at the differential first-order equation:

$$Z' = -Z^2 - \sqrt{Z}.$$

The solution of the initial equation is fulfilled after this only by means of the quadratures and taking the logarithm.

The following conditions must be carried out: when $\eta=0$ (axis of jet), v=0, i.e., $F \doteq e^{z}=0.$ Since u~F'=z'• e^{z} when $\eta=0$ does not become zero, then z' when $\eta=0$ is equal to infinity of the same order as as $\frac{1}{e^{z}}$. With the changed scale for $\eta_0=0$ we obtain also:

$$F=0 \quad (15a)$$

and

$$F'=1. \quad (15b)$$

Thus, are established/installed two conditions for z which are sufficient for the differential second order equation. From the boundary condition u=0, i.e.

$$z'=Z=0 \quad (16)$$

for boundary η is determined value η_{zp}.

From equation (14) it is obtained after the integration:

$$\eta=c-\frac{2}{3}\cdot\left\{\ln(\sqrt{Z}+1)-\ln[(Z-\sqrt{Z}+1)^{1/2}]+\sqrt{3}\cdot\operatorname{arctg}\frac{2\sqrt{Z}-1}{\sqrt{3}}\right\}. \quad (17)$$

from condition z'=Z=∞ for $\eta=0$ we find integration constant c:

$$0=c-\frac{2}{3}\cdot\sqrt{3}\cdot\frac{\pi}{2},$$

whence

$$c=\frac{\pi}{\sqrt{3}}.$$

Condition (16) Z=0 when η_2 it gives:

$$\eta_2=c-\frac{2}{3}\cdot\sqrt{3}\cdot\operatorname{arctg}\left(-\frac{1}{\sqrt{3}}\right)=2,412.$$

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In order to completely satisfy condition (15), it is necessary to investigate the behavior of differential equation (14) when $\eta = \infty, Z = 0$.

Since equation (17) in this region is not applicable, then let us form an equation in the new form, suitable for $\eta = 0, Z = \infty$. It is obvious that with $Z \to \infty$:

$$\frac{dZ}{d\eta} \to -Z^2,$$

i.e.

$$Z = z' \to \frac{1}{\eta}.$$

Consequently with $\eta = 0$

$$z \to \ln \eta + c_1$$

and

$$F = z' \cdot e^z = e^{c_1}.$$

From condition (15b) we find in this case latter/last integration constant $c_1=0$. On the basis of the given modification, we obtain as the asymptotic approximation/approach when $\eta = 0$:

$$z' = \frac{1}{\eta} - 0{,}4 \cdot 1\,\eta + 0{,}01 \cdot \eta^2 + \ldots$$

and

$$z = \ln \eta - \frac{0{,}8}{3} \cdot \eta^2 + \frac{0{,}01}{3} \cdot \eta^3 + \ldots \qquad (18)$$

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The quality of asymptotic approximation/approach (18) it is easy to judge by means of its comparison with precise equation (17) in that region, in which both of forms of equation are suitable. The way of calculation following: they calculate according to formula (17) $\eta(Z)$ and at the same time $Z(\eta)=z'(\eta)$ and they obtain hence, for example, by graphic integration, $z(\eta)$. Moreover in the region near $\eta=0$, $z'=\infty$ is utilized above obtained asymptotic approximation/approach.

Hence by taking the logarithm and multiplication we find $F=e^{z}$ and $F'=z'\cdot e^{z}$.

III. Expansion of jet as the problem of axial symmetry.

The corresponding problem with the axial symmetry in which also the jet escape/ensues from the very eyelet within the wall, it is permitted by accurately the same method as two-dimensional problem. Assuming that the pressure in the jet is constant, then:

$$2\cdot\pi\cdot\int_{-\infty}^{+\infty} u^2\cdot y\cdot dy=\text{const},$$

whence we obtain expression for u:

After assuming

$$u=\frac{1}{x}\cdot f(\eta).$$

$$\int f(\eta)\cdot\eta\cdot d\eta=F(\eta),$$

let us find

$$u=\frac{F'}{x\cdot\eta}$$

and

$$v=\frac{F'}{x}-\frac{F}{x\cdot\eta}.$$

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Exactly as in two-dimensional problem, on the basis of the theorem of momentum or integrating equation of motion, we obtain the differential equation:

$$c^2\cdot\left(F''-\frac{F'}{\eta}\right)^2=F\cdot F'. \qquad (19)$$

Introducing the changed scale for η, we simplify this differential equation:

$$\left(F''-\frac{F'}{\eta}\right)^2=F\cdot F'. \qquad (19a)$$

By means of the replacement

$$z=\ln F,\ F=e^z,$$

concluded

$$\left(z''+(z')^2-\frac{z'}{\eta}\right)^2=z',$$

and finally, introducing $Z=z'$, we obtain differential first-order equation:

$$Z'=\frac{Z}{\eta}-Z^2-\sqrt{Z}. \qquad (20)$$

In this case we have available the following conditions with $\eta=0$: u does not disappear, when v=0; i.e. $F(0)=e^{z(0)}=0$, while $\frac{F'}{\eta}=\frac{z'\cdot e^z}{\eta}$ remains finite quantity, also, with the changed scale to equal 1.

Satisfying these conditions, it is possible $z(\eta)$ near $\eta=0$ to present in the form of series/row. In order when $\eta=0: e^z=0$, z must be equal to negative ∞ and besides to the same degree as $\ln \eta^2$; in this case $\frac{F'}{\eta}$ acquires finite value. Because of this we obtain the following series/row in which adjacent terms differ on $\eta^{3/2}$:

$$Z = \frac{2}{\eta} + a \cdot \sqrt{\eta} + b \cdot \eta^2 + c \cdot \eta^{7/2} + d \cdot \eta^5 + e \cdot \eta^{13/2} \ldots \tag{21}$$

Coefficients are established/installed by means of the substitution of this solution in the differential equation and the comparison of terms with the equal degrees:

$$a = -\frac{2}{7} \cdot \sqrt{2}; \quad b = -\frac{1}{245}; \quad c = \frac{\sqrt{2}}{1715}; \quad d = \frac{37}{240\,100}; \quad e = 0{,}000014 \ldots$$

The convergence of this series/row near the jet boundary $\eta_{гр} (z=0)$ very bad; however, it is possible to give such modification which is conveniently applied in region $\eta_{гр}$.

Let

$$\bar{\eta} = \eta_{гр} - \eta$$

and

$$z = \bar{a} \cdot \bar{\eta}^2 + \bar{b} \cdot \bar{\eta}^3 + \bar{c} \cdot \bar{\eta}^4 + \bar{d} \cdot \bar{\eta}^5 + \bar{e} \cdot \bar{\eta}^6 + \bar{f} \cdot \bar{\eta}^7 + \ldots$$

then

$$\bar{a} = \frac{1}{4}; \quad \bar{b} = -\frac{1}{8 \cdot \eta_{гр}}; \quad \bar{c} = -\frac{3}{64 \eta_{гр}^2};$$

$$\bar{d} = \frac{1}{64} - \frac{3}{128 \cdot \eta_{гр}^3}; \quad \bar{e} = -\frac{19}{256 \cdot 5 \cdot \eta_{гр}} - \frac{133}{256 \cdot 40 \cdot \eta_{гр}^4};$$

$$\bar{f} = -\frac{0{,}00278}{\eta_{гр}^2} + \frac{\ldots}{\eta_{гр}^5} \ldots$$

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Integration constant η_{kp} can be determined via comparison at the fixed point of values z which are known of both of solutions.

There is obtained

$$\eta_{kp} = 3,4.$$

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